Trade-offs in Compensating Transfers for the Model of Occupational Choice

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Abstract
Using the model of occupational choice, we study the trade-offs faced by a benevolent government that aims Pareto improvement from an economic environment change via incentive compatible compensating transfers. When the transfers are designed after the shock is realized, then the trade-off is between Pareto improvement and overcompensation. When the agents anticipate the future transfer schemes, then the trade-off is between the size of the aggregate production gains and the amount of overcompensation.

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1 Introduction

There are many instances in the economy where potentially Pareto improving environmental changes are blocked by the opposition by some groups that will be harmed by such changes. For example, trade liberalization for a small open economy often encounters severe political oppositions within its territory despite the fact that such liberalization usually brings about aggregate level gains to the economy as a whole. If the economic change that brings aggregate gains were really a Pareto improving one, then there should be nobody who opposes to such a change. In the real world, however, such a change will bring about both winners and losers and the “actual” Pareto improvement requires the income redistribution because such an improvement is merely a “potential” one. Why do some people clamour against such potentially Pareto improving change? In most cases, it is because actual income transfers are seldom executed by the government after the change has occurred. The compensating redistribution seldom takes place and losers are often left untreated. Even if some redistribution schemes are carried out, they are never done in full. The lack of satisfactory redistributing transfers is the main reason why a potential Pareto improving change has so many opponents to the potentially beneficially changes.

Why and how is compensating redistribution unsatisfactory? This paper tries to understand the reasons why the governmental compensation program often fails.

In fact, there are two strands of criticisms about the compensating redistribution schemes. The first critics say that the compensation coverage is not sufficient. In other words, there are losers who do not get enough (or adequate) compensating transfers. The current compensation schemes is said to be imperfect in the sense that its coverage and the amount is insufficient compared to the actual losses from the economic change. Some of the losers from the change are left untreated. Even if they are compensated, the amount is usually regarded as smaller than their actual losses. The second critics say that the existing compensating transfers may be overcompensating (compared with the intended consequences) and the money is wasted because many transfers reach those who are not originally targeted. This problem of overcompensation requires some explanation. It is true that there exist some attempts to compensate the losers from some change. For example, there exists the Trade Adjustment Assistance Program in the United States which provides more generous unemployment insurance for those who lost jobs from trade than for those who lost jobs for other reasons. In any such schemes that aim to compensate losers, there are always the targeted group of individuals and the targeted amount. The problem of such schemes is that, in many cases of the actual transfers, the transfer amount tends to be larger than what was originally intended and that the some of the money is received by someone to whom the government did not intend to give the money from the beginning.\footnote{Here I am not talking about the illegal actions such as fraud. There are instances that some people get the subsidies from the government legally, but the government did not target them originally.}

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1 Here I am not talking about the illegal actions such as fraud. There are instances that some people get the subsidies from the government legally, but the government did not target them originally.
If we look at the surface of these two critics, they seem rather contradictory because the first seems to say that the compensation is not enough while the second seems to say that the compensation is too much. The purpose of this paper is to formalize one possible explanation of these seeming paradoxes. Imagine that an economy experiences a change that brings about aggregate gains to the economy as a whole. Any such change will bring about distributional consequences and create gainers and losers. The government aims Pareto improvement by subsidizing the losers with the money collected from taxes from the gainers. Nevertheless, the lack of information about individuals and the limits on instruments of tax system may prevent the government from properly identifying who are the gainers and who are the losers. This lack of complete identification may cause the seemingly contradictory paradoxes. If the government aims to help every single loser in the economy, it may end up giving the subsidy to the gainers who did not need to be helped. This may cause the problem of overcompensation and may use up the governmental budget to help the losers. If the overcompensation problem is severe, then the government may not be able to balance the budget of the compensation scheme alone. In such a case, the policy maker may give up carrying out the compensating transfers at all. This may cause an insufficient coverage of the compensation for losers.

The explanation utilizes the idea of Roy (1951) model of occupational choice from labor economics. I combine Roy model with the framework provided by Ruffin (2001) whose model uses technology with both quasi-specific and regular factors of production. So in this paper, individual agents in the economy are endowed with a bundle of multi-dimensional skills, together with other regular types of productive factors. The compensation with regard to the regular factors can be taken care of by the commodity taxation scheme proposed in Dixit and Norman (1980, 1986). However, the design of compensation scheme with regard to the multi-dimensional vector of skills is not straightforward. Multi-dimensional human capital skills are embodied into workers and they cannot be sold in the market separately from the workers themselves. Employers must buy the set of bundles of skills when they hire a worker. The importance of bundling restriction was noted in Murphy (1986) and in Ohnsorge and Trefler (2007). Workers sort themselves into the jobs that pay the most given the set of skills they have. Among the skills the workers have, all skills except one become unused latent talents which do not have market values. Those unused talents serve workers as second and third best alternatives when they decide to choose their sector to work in. The latent skill which has the highest return serve the worker as the opportunity cost of the current job the worker has. It is inability of the government to capture the exact size of this opportunity cost of individual agents that prevent it from designing the Pareto improving taxation scheme which is free of trade-offs.

The paper is organized as follows. Section 2 develops the basic model of occupational choice with heterogeneous agents with multi-dimensional skills. It

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2 Quasi-specific factors in Ruffin (2001) are similar to the multi-dimensional vector of skills in this paper. Their differences will be discussed in Section 6.
is a two sector model produced by one general factor and a vector of occupational abilities. Section 3 looks at the welfare analysis of individual agents when there is no compensating distribution. Section 4 investigates various desirable and undesirable properties of the compensating redistribution schemes. Section 5 will discuss the trade-offs faced by the government when it tries to carry out the compensating redistribution schemes. The final section summarizes the results and suggests some possible extensions.

The model in this paper makes no pretense of generality. It relies on extremely restrictive assumptions of technology and utility. Nonetheless, it is to be hoped that the paper provides some useful insights into some aspects of trade-offs faced by the government when it must carry out compensating transfers after some economic change. The setup of the model starts out from the one of a small open economy. However, the basic workings of the model carry through to any economic changes occurred in the closed domestic economy as well. The main reason why we adopted the case of a small open economy is to conserve the space\(^3\) and to concentrate readers’ attention on the main topic of this paper, namely the analysis of the trade-offs of compensating redistribution schemes.

\section{The Basic Model}

Consider a small open economy that produces two outputs \(X\) and \(Y\) whose market prices are denoted as \(P_X\) and \(P_Y\). Two output goods are produced by combining two types of input factors held by each individual: multi-dimensional occupation specific talents (abilities) and generic factors.

The economy comprises a continuum of heterogeneous atomless agents whose measure can be normalized to unity. An agent is characterized by a vector of two dimensional occupational abilities \((\theta, \tau)\) which is jointly distributed by \(F(\theta, \tau)\) whose density is written \(f(\theta, \tau) > 0\) everywhere over a compact and convex type space \(\Theta \subset \mathbb{R}^2\). Each component of a vector \((\theta, \tau)\) represents the size of occupation specific talent. The size of \(\theta\) (respectively, \(\tau\)) corresponds to the agent’s ability to produce output \(X\) (respectively, \(Y\)). An agent \((\theta, \tau)\) is also endowed with \(K(\theta, \tau) \geq 0\) units of generic factors of production whose distribution we do not specify other than that the total amount available in the economy is written as \(\bar{K}\), i.e.,

\[
\int_{(\theta, \tau) \in \Theta} K(\theta, \tau) dF(\theta, \tau) = \bar{K}
\]  

holds. Hence, the total factor endowments held by an agent \((\theta, \tau)\) is written as the vector \((\theta, \tau; K(\theta, \tau))\).

\(^3\)The benefit of the setup of a small open economy is to make the analysis easy because the change in the economy appears as the change in the relative price of outputs (terms of trade). Otherwise, the change in output prices may come from either the change in technology, endowments, or preferences. In either cases we need to analyze the welfare change from both price and technology (endowments, or preferences) changes.
There are several important assumptions about agents and factors of production which may be quite different from regular models. Here we summarize these assumptions as A1 - A3:

**A1: Skill Specificity** An occupation specific talent is specific to a particular sector. Its marginal product is positive in the specific sector but zero in the other sector. In order to produce output $X$ (respectively, $Y$), an agent must use both generic factor $K$ (or $k$) and an occupation specific talent $\theta$ (respectively, $\tau$). Although general factor can be used in either sector, an occupation specific talent $\theta$ (respectively, $\tau$) has no value in production of $Y$ (respectively, $X$).

**A2: Single Job (Skill Bundling)** An agent can choose only one job at a time. Because human capital skills are embodied in an agent, each component of skills cannot be sold separately. (See Murphy 1986.) In other words, a skill vector $(\theta, \tau)$ must be sold as a package. Similarly to the setup in Ohnsorge and Treffer (2007), a bundle of skills cannot be decomposed. When an agent uses one component of the skill vector, the other component merely represents a latent skill which is just a potential one.

**A3: No Market for Skills** There does not exist any market for occupation specific talents. While there is a market for generic factor $K$ so that agents can sell and buy a portion of their endowments, agents cannot sell portion of their talents. According to the magazine article in *The Economist* (March 26, 1994), the capital markets for human capital investment may be imperfect. The magazine article says “For instance, borrowing to finance an investment in human capital may be difficult because would-be trainees lack collateral, or because the costs of administration and collection make such loans unattractive to private lenders.” So assume there is no market for skills.

Because there is a market for generic factor $K$, the price of the factor is denoted as $r$. An agent with skill vector $(\theta, \tau)$ is assumed to earn the residual profit for his used skill. An agent in this model is a residual claimant for his used skill.

Now that we described the assumptions about agents and factors of production, we now explain technology and supply side of the economy.

### 2.1 Technology and Supply Side

All economic agents share the same constant returns to scale technology. Both goods are produced with symmetric Cobb-Douglas production functions with a parameter $a \in (0, 1)$ which represents a share of general factor income:

$$
\begin{align*}
  x(\theta, \tau) &= (kX)^a \cdot (\theta)^{1-a} \\
  y(\theta, \tau) &= (kY)^a \cdot (\tau)^{1-a}
\end{align*}
$$

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4I use capital letter to indicate endowment and lower case to indicate employment.
where \( x(\theta, \tau) \) and \( y(\theta, \tau) \) are the potential amounts of production of each good by an agent with skill vector \((\theta, \tau)\), and \( k_X \) and \( k_Y \) are the quantity of general factor \( K \) used for the production. Note that an agent \((\theta, \tau)\) in (2) does not actually produce both \( x(\theta, \tau) \) and \( y(\theta, \tau) \) but that he produces either.

As a pricetaker in the markets for outputs and general factors, an agent take a price vector \((P_X, P_Y, r)\) as given and try to maximize his residual claims. The agent’s profit functions are the solutions to the following problems:

\[
\begin{align*}
\pi_X(\theta, \tau; P_X, P_Y, r) &= \max_{k_X} \{P_X \cdot x(\theta, \tau) - r \cdot k_X\} \\
\pi_Y(\theta, \tau; P_X, P_Y, r) &= \max_{k_Y} \{P_Y \cdot y(\theta, \tau) - r \cdot k_Y\} \\
\end{align*}
\]

Note that an agent \((\theta, \tau)\) will calculate both problems in his head but in the end he chooses only one sector to work so only either \(\pi_X(\theta, \tau; P_X, P_Y, r)\) or \(\pi_Y(\theta, \tau; P_X, P_Y, r)\) matters for an agent. Using the Cobb-Douglas production functions given by (2), actual values of the profit functions are given as

\[
\begin{align*}
\pi_X(\theta, \tau; P_X, P_Y, r) &= (P_X)^{\frac{1}{\alpha_X}} \cdot (r)^{\frac{\alpha_X}{\alpha_X + \alpha_Y}} \cdot \left( a^{\frac{\alpha_Y}{\alpha_X + \alpha_Y}} - a^{\frac{\alpha_X}{\alpha_X + \alpha_Y}} \right) \cdot \theta \\
\pi_Y(\theta, \tau; P_X, P_Y, r) &= (P_Y)^{\frac{1}{\alpha_Y}} \cdot (r)^{\frac{\alpha_Y}{\alpha_X + \alpha_Y}} \cdot \left( a^{\frac{\alpha_X}{\alpha_X + \alpha_Y}} - a^{\frac{\alpha_Y}{\alpha_X + \alpha_Y}} \right) \cdot \tau \\
\end{align*}
\]

(4)

By comparing the two (potential) profit functions in (4), the type space can be divided into two partition groups: The group \(\Theta_X\) of sector \(X\) producers and the group \(\Theta_Y\) of sector \(Y\) producers, i.e.,

\[
\begin{align*}
\Theta_X &= \{ (\theta, \tau) \in \Theta : \tau < (P_X/P_Y)^{\frac{1}{\alpha_X}} \cdot \theta \} \\
\Theta_Y &= \{ (\theta, \tau) \in \Theta : \tau > (P_X/P_Y)^{\frac{1}{\alpha_Y}} \cdot \theta \} \\
\end{align*}
\]

(5)

where the division of labor line is written as

\[
\tau = (P_X/P_Y)^{\frac{1}{\alpha_X}} \cdot \theta
\]

(6)

above which agents produce \(Y\) and below which agents produce \(X\). The assignment of workers to particular sectors is done here by comparative advantage of individual agents as in Sattinger (1975).

The output prices \((P_X, P_Y)\) in the regular economic models will usually be determined by both supply and demand conditions in the equilibrium. However, this paper considers a change in \((P_X, P_Y)\) to be given rather exogenously for the sake of simplicity.

### 2.2 Demand Side

Consumers take market prices \((P_X, P_Y, r)\) as given. For an agent with the skill vector \((\theta, \tau)\) with his utility function given as \(u(c_X, c_Y)\), the profit maximization problem can be written as follows:

\[
\max_{c_X, c_Y} u(c_X, c_Y) \quad \text{s.t.} \quad P_X \cdot c_X + P_Y \cdot c_Y \leq I(\theta, \tau)
\]

(7)
In order to conserve the notation and space, we use \((c_X, c_Y)\) in stead of explicitly writing \((c_X(\theta, \tau), c_Y(\theta, \tau))\) to indicate the vector of consumption of two goods by a particular agent \((\theta, \tau)\). The total income \(I(\theta, \tau)\) of an individual \((\theta, \tau)\) in equation (7) must come from the market value of all the possessed factors of production including the residual profits from his talent skills, which can be written as

\[
I(\theta, \tau) = r \cdot K(\theta, \tau) + \max\{\pi_X(\theta, \tau; P_X, P_Y, r), \pi_Y(\theta, \tau; P_X, P_Y, r)\}
\]

where the size of residual profit is determined by the agent’s occupational choice.\(^5\)

In general, utility function of the individual agents shall be homothetic, monotonically increasing, strictly quasi-concave and twice-continuously differentiable function, but here I assume a specific functional form in order to make the following analysis tractable. Assume that all the individuals have identical, Cobb-Douglas preferences, spending half of their income on either good.\(^6\) Now we can utilize a convenient way of price normalization, i.e., the geometric mean of two output prices to be one. Use a relative price parameter \(p > 0\) so that \(P_X = p\) and \(P_Y = 1/p\) hold. This way we can have the value of consumer price index to be fixed given any relative price change and we can compare the welfare of different states just by looking at the income expressed using the parameter \(p\) in order to focus on the substitution effects in consumption.

2.3 Factor Market Equilibrium

In order to derive the equilibrium condition for the factor market, consider the full employment condition for general factors taking into account the occupational choices of the individual agents given by the job-assignment partitions of (5):

\[
\int_{\Omega_X} k_X(\theta, \tau) dF(\theta, \tau) + \int_{\Omega_Y} k_Y(\theta, \tau) dF(\theta, \tau) = K
\]

where \(k_X(\theta, \tau)\) and \(k_Y(\theta, \tau)\) are the quantity of general factor used (employed) in the actual production of each output. If we plug in the solutions \(k_X(\theta, \tau)\) and \(k_Y(\theta, \tau)\) for the problem (3) using the production function (2) for each agent \((\theta, \tau)\), then we can obtain the equilibrium equation for the factor price \(r\):

\[
r = a \cdot K^{-a-1} \left[ \int_{\Omega_X} \theta dF(\theta, \tau) + \int_{\Omega_Y} \tau dF(\theta, \tau) \right]^{1-a} \]  

\(^5\)If \((\theta, \tau) \in \Theta_X\), then \(I(\theta, \tau) = r \cdot K(\theta, \tau) + \pi_X(\theta, \tau; P_X, P_Y, r)\) and if \((\theta, \tau) \in \Theta_Y\), then \(I(\theta, \tau) = r \cdot K(\theta, \tau) + \pi_Y(\theta, \tau; P_X, P_Y, r)\).

\(^6\)The utility function can be written as \(u(c_X, c_Y) = 2\sqrt{c_X \cdot c_Y}\) and its indirect utility function is \(v(P_X, P_Y, I) = I/\sqrt{P_X P_Y}\).
where the size of factor price depends on parameters, output prices and the size
of aggregated skill levels which can be noted as

\[
\begin{align*}
\int_{\Theta^X} \theta dF(\theta, \tau) & \equiv V_{\theta}(p) \\
\int_{\Theta^Y} \tau dF(\theta, \tau) & \equiv V_{\tau}(p)
\end{align*}
\]  

(11)

where \(V_{\theta}'(p) > 0\) and \(V_{\tau}'(p) < 0\) holds true. (The proof is in the appendix.)

Let us define the following notation.

\[
s(p) \equiv p^{\frac{1}{1-a}} \cdot V_{\theta}(p) + p^{\frac{1}{1-a}} \cdot V_{\tau}(p)
\]  

(12)

The reason why we can write \(s(\cdot)\) as a function of parameter \(p\) is because both \(V_{\theta}\) and \(V_{\tau}\) in (11) depend on \(p\).

Using (12), we can rewrite the factor price equation (10) as a function of \(p\):

\[
r(p) = a \cdot K^{a-1} \cdot s(p)
\]  

(13)

where \(a\) and \(K\) are parameters.

### 2.4 Equivalence of National Income with GNP

Equilibrium national income can also be expressed as a function of relative output prices, \(p\).

\[
GNI(p) \equiv r(p) \cdot K + \int_{\Theta^X} \pi_X(\cdot) dF(\theta, \tau) + \int_{\Theta^Y} \pi_Y(\cdot) dF(\theta, \tau)
\]  

(14)

I now present an intermediate result relating to the relationship between national income and generic-factor income.

**Lemma 1** Generic-factor income is proportional to national income, as expressed by the following equation.

\[
r(p) \cdot K = a \cdot GNI(p)
\]  

(15)

This follows directly from equations (8), (13) and (14). This proportional relationship in (15) arises because the production functions for the two sectors are Cobb-Douglas and symmetric. The proof is in the Appendix.

Note also that national factor income is equal to the gross national product.

\[
GNI(p) = GNP \equiv P_X \cdot \int_{\Theta^X} x(\theta, \tau) dF(\theta, \tau) + P_Y \cdot \int_{\Theta^Y} y(\theta, \tau) dF(\theta, \tau)
\]  

(16)

It can be easily shown that the relationship in (15) is consistent with (16).
2.5 Goods Market Equilibrium

Now I analyze goods-market equilibrium. We are interested in two equilibria: one for free trade and one for autarky. I investigate trade volumes for the trading equilibrium and derive the goods-market-clearing conditions for the autarky equilibrium.

2.5.1 Trading equilibrium

A trading equilibrium is represented by a net import vector, \( m(p) \), for a given relative price, \( p \): \[
m(p) \equiv (ED_X(p), ED_Y(p)) = (C_X(p) - X(p), C_Y(p) - Y(p)),
\]
where \( ED_X(p) \) and \( ED_Y(p) \) are the excess demand functions for sectors \( X \) and \( Y \), respectively, and \( C_X(p) \) and \( C_Y(p) \) represent aggregate demand for \( X \) and \( Y \): \[
C_X(p) \equiv \int_{\Theta} c_X dF(\theta, \tau) \quad \text{and} \quad C_Y(p) \equiv \int_{\Theta} c_Y dF(\theta, \tau).
\]
In addition, \( X(p) \) and \( Y(p) \) represent aggregate supply for \( X \) and \( Y \): \[
X(p) \equiv \int_{\Theta_X} x(\theta, \tau) dF(\theta, \tau) \quad \text{and} \quad Y(p) \equiv \int_{\Theta_Y} y(\theta, \tau) dF(\theta, \tau).
\]

2.5.2 Autarky

Autarky is a special case in which the autarky price, \( p^A \), makes \( m(p^A) = 0 \). I now derive the conditions for the autarky equilibrium. Given the utility function \( u(c_X, c_Y) = 2\sqrt{c_X \cdot c_Y} \), the aggregated Walrasian-demand functions for goods \( X \) and \( Y \) can be written respectively as \[
\begin{align*}
C_X(p) &= \frac{GNI(p)}{2} \\
C_Y(p) &= \frac{pGNI(p)}{2}
\end{align*}
\]
where the left panel shows the individual demand functions and the right panel shows the market demand functions. By using the previous results, I express the aggregate production in terms of \( p \), as follows.

Thus, given the result in (15), when \( p = p^A \), the following equations hold.

\[\begin{align*}
V_\theta(p) &= \frac{K}{2} \cdot \left( \frac{r(p)}{a} \right)^{\frac{1}{\alpha}} \\
V_\tau(p) &= \frac{K}{2} \cdot \left( \frac{p_r(p)}{a} \right)^{\frac{1}{\alpha}}
\end{align*}\]
Substituting the equilibrium generic-factor return (13) into (17) yields the following autarky condition for aggregate employment of the specific occupational factors.

\[ p^{\frac{1}{1-a}} \cdot V_\theta(p) = p^{-\frac{1}{1-a}} \cdot V_r(p) \mid_{p=p^A} \]  

(18)

Under autarky, \( p = p^A \), as expressed in equation (18).

3 Welfare Analysis of Individual Agents

This section tries to look at the effects of some exogenous economic shock to the welfare of individual agents. Throughout this section, I rule out the possible compensation transfer schemes. Therefore, the welfare analysis is the one before the introduction of any transfer schemes by the government.

The following analysis can be applied to any changes in economic variables, but in order to make analysis tractable I will focus on the specific shock to the economy, namely, the change in terms of trade for a small open economy. The analysis of small open economy is easy because the change in relative output price comes from outside of the model. In order to analyze the welfare change of the individual agents, the setting of a small economy saves many unnecessary steps. The essence of the following analysis can be carried over to the other cases of technological changes or demand changes, except for the necessary adjustments that must be made in the welfare analysis.

Let us now start the analysis by defining the exogenous shock to the economy as rising relative price of good \( X \).

**Definition 1** Before shock price parameter was \( p^0 \) and after shock parameter becomes \( p^1 > p^0 \).

After this shock, the small open economy will export \( X \) and import \( Y \). Now by using the price parameter \( p \) the division of labor line (6) can be rewritten as

\[ \tau = p^{\frac{1}{1-a}} \cdot \theta \]

which will have steeper slope after the shock because \( p^1 > p^0 \) and \( \frac{2}{1-a} > 1 \).

Following the partitioning of the type space given by (5), let \( \Theta_X(p^0) \) and \( \Theta_Y(p^0) \) represent the partitions of type space into the one for sector \( X \) produces and for \( Y \) producers **ex ante**, and let \( \Theta_X(p^1) \) and \( \Theta_Y(p^1) \) represent each partition **ex post**. Then the whole type space \( \Theta \) can be divided into the following 3 partitions:

1. Job stayers in sector \( X \) (workers who work in sector \( X \) ex ante and ex post):

\[ \Theta_{XX} \equiv \Theta_X(p^0) \cap \Theta_X(p^1) \]

2. Job switchers who moved from sector \( Y \) to \( X \) (workers who work in sector \( Y \) ex ante and in sector \( X \) ex post):

\[ \Theta_{YX} \equiv \Theta_Y(p^0) \cap \Theta_X(p^1) \]

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3. Job stayers in sector \( Y \) (workers who work in sector \( Y \) ex ante and ex post):

\[
\Theta_{YY} \equiv \Theta_Y(p^0) \cap \Theta_Y(p^1)
\]

Since \( p^1 > p^0 \), there does not exist job switchers in the other direction because \( \Theta_X(p^0) \subset \Theta_X(p^1) \) and \( \Theta_Y(p^0) \supset \Theta_Y(p^1) \) implies \( \Theta_X(p^0) \cap \Theta_Y(p^1) = \emptyset \).

I can now summarize the results concerning the welfare changes of the individual agents.

**Proposition 1** The economic welfare of the job staying agents will rise by the increase in the price of the goods which they are producing and will drop by the decrease of the price of their goods.

This proposition says that the welfare change for the job staying agents are the same as the analysis for specific factor owners in the specific factors model of international trade.

**Proof.** When the price of \( X \) is given as \( p \), the profit for an agent \((\theta, \tau) \in \Theta_X(p)\) will be written as

\[
\pi_X(\theta, \tau; p) = \left[ p^{\frac{1}{a^2}} \cdot (r(p))^{\frac{1}{1-a}} \cdot \left( a^{\frac{1}{1-a}} - a^{\frac{1}{1-a}} \right) \right] \cdot \theta. \tag{19}
\]

The equilibrium general factor price \( r \) can be written using \( p \),

\[
r(p) = a \cdot \overline{K}^{1-a} \cdot [s(p)]^{1-a} \tag{20}
\]

which can be substituted into the equation (19), and we get

\[
\pi_X(\theta, \tau; p) = \overline{K}^{a(1-a)} \cdot p^{\frac{1}{1-a}} \cdot [s(p)]^{-a} \cdot \theta \tag{21}
\]

Because both \( \overline{K}^{a(1-a)} \) and \( \theta \) are non negative, the sign of the derivative of \( p^{\frac{1}{1-a}} \cdot [s(p)]^{-a} \) with respect to \( p \)

\[
\frac{d}{dp} \left( p^{\frac{1}{1-a}} \cdot [s(p)]^{-a} \right) = s^{-a} \cdot p^{\frac{1}{1-a}} \cdot a \cdot \left( \frac{1}{a(1-a)} - \frac{p \cdot s'(p)}{s(p)} \right)
\]

will be the same as the sign of the derivative of the profit for an agent. By assumption, \( 0 < a < 1 \) and \( p > 0 \),

\[
s^{-a} \cdot p^{\frac{1}{1-a}} \cdot a > 0 \text{ and } \frac{1}{a(1-a)} > 0
\]

are clear. Also, when \( p > p^0 \) holds \( s'(p) < 0 \) must also hold true. Therefore, we can say that

\[
\left( \frac{1}{a(1-a)} - \frac{p \cdot s'(p)}{s(p)} \right) > 0.
\]
This proves that job stayer in sector $X$ will gain from increase in price $p$. The analysis for the sector $Y$ job stayers can be conducted in a similar manner which is omitted here.

Next, let us look at the welfare changes for job switching agents.

**Proposition 2** Among those who are forced to switch jobs due to some economic shocks, there exist both welfare gainers and losers. Their sizes of gain or loss depend on the comparative advantages of the individual agents, namely, the relative sizes of the components of their skill vectors.

The result of this proposition sounds very interesting because it may look contradictory to the popular belief about the relationship between the economic shock and the job losses. We tend to think that job losers (those who are forced to switch) should all be losers in their welfare, but the analysis shows that there exist welfare gainers as well as losers. Furthermore, the analysis predicts that the amount of gain or loss depends on the parameter of comparative advantage for an individual.

**Proof.** When the relative price parameter is $p$, then the value of the profit function for agent $(\theta, \tau)$ if the agent works for sector $Y$ is written as

$$\pi_Y(\theta, \tau; p) = \left[ p^{\frac{1}{a}} \cdot (r(p))^{\frac{1}{1-a}} \cdot \left( a^{\frac{1}{1-a}} - a^{\frac{1}{a}} \right) \right] \cdot \tau. \quad (22)$$

By using (20), we can rewrite (22) as

$$\pi_Y(\theta, \tau; p) = K(1-a) \cdot p^{\frac{1}{1-a}} [s(p)]^{-a} \cdot \tau. \quad (23)$$

The profit of job switchers $(\theta, \tau) \in \Theta_{YX}$ was $\pi_Y(\theta, \tau; p^0)$ ex ante and is $\pi_X(\theta, \tau; p^1)$ ex post. Therefore, the percentage change of welfare for the job switchers can be written as

$$\% \Delta \pi \equiv \frac{\pi_X(\theta, \tau; p^1) - \pi_Y(\theta, \tau; p^0)}{\pi_Y(\theta, \tau; p^0)} = \frac{(p^1)^{\frac{1}{1-a}} [s(p^1)]^{-a}}{(p^0)^{\frac{1}{1-a}} [s(p^0)]^{-a}} \cdot \frac{\theta}{\tau} - 1. \quad (24)$$

The equation (24) can be thought of as an affine function of comparative advantage parameter $\theta/\tau$ because the output prices $p^0$ and $p^1$ are given exogenously to the model. In order to prove the proposition, we take three steps:

1. Consider first the job switching agents near the ex ante division of labor line: $\tau = (p^0)^{\frac{1}{1-a}} \cdot \theta$. Agents on this line must have been indifferent between working in either sector ex ante, thus $\pi_X(\theta, \tau; p^0) = \pi_Y(\theta, \tau; p^0)$. Therefore, the agents must have gained exactly the same percentage as job staying agents in sector $X$ from Proposition 1.
2. Consider the job switching agents near the ex post division of labor line: \( \tau = (p^1)^{\frac{1}{1+a}} \cdot \theta \). Agents on this line must now be indifferent between working in either sector ex post, thus \( \pi_X(\theta, \tau; p^1) = \pi_Y(\theta, \tau; p^1) \). Therefore, the agents must have lost exactly the same percentage as job staying agents in sector Y from Proposition 1.

3. From step 1 and 2, we know that the welfare changes take positive value near the ex ante division of labor line and negative value near the ex post division of labor line. Because the affine function (24) is a continuous function of the parameter \( \theta/\tau \), there exists the value of \( \theta/\tau \) such that the welfare change \( \% \Delta \pi = 0 \) must hold. (Intermediate value theorem) In fact, when the following relationship holds, welfare gain becomes zero.

\[
\theta/\tau = (p^0)^{\frac{1}{1+a}} [s(p^0)]^{-a} / (p^1)^{\frac{1}{1+a}} [s(p^1)]^{-a}
\]  
(25)

By rearranging the terms in (25), we can write the gain-zero line in the type space:

\[
\tau = (p^1)^{\frac{1}{1+a}} [s(p^1)]^{-a} / (p^0)^{\frac{1}{1+a}} [s(p^0)]^{-a} \cdot \theta
\]

above which agents are gainers and below which agents are losers.

This third step concludes the proof. ■

The percentage change in welfare in (24) shows that it takes the same value along the rays from origin. The fortune and misfortune of job movers will change along the slope of the rays from the origin. The steeper the slope on which agents locate is, the smaller the gains or the larger the losses become. If we look at the agents near the ex ante division of labor line, \( \tau = (p^0)^{\frac{1}{1+a}} \cdot \theta \), we know that the welfare of these agents should be the same as the job staying agents for sector X. When we observe the steeper slope of the rays from the origin from there up to the gain zero line, \( \tau = \left( (p^1)^{\frac{1}{1+a}} [s(p^1)]^{-a} / (p^0)^{\frac{1}{1+a}} [s(p^0)]^{-a} \right) \cdot \theta \), we can say that the percentage change in the welfare gain decreases. For those who are on the gain zero line, the rate of welfare change is zero. If the slope is steeper that the gain zero line and below the ex post division of labor line, \( \tau = (p^1)^{\frac{1}{1+a}} \cdot \theta \), the welfare of the agents gets worsened. The degree of worsening becomes severe when the slope is steeper. For the agents near the ex post division of labor line, the welfare decreases as much as those who stayed in the losing sector Y. Therefore, if we were to draw the line of “iso-percentage change of welfare”, then the lines should coincide with the rays from the origin.

4 Trade-offs in Compensation Schemes

The previous section looked at the welfare changes of individual agents when there is no compensating transfer scheme imposed by a benevolent government.
In this section, we analyze how it constructs an optimal compensation scheme. The results of the preceding analysis have shown that there are both winners and losers among job switchers. Having analyzed the effect of a terms-of-trade change without compensation, next, I consider a government redistribution policy that aims to achieve a Pareto improvement (from trade) and a balanced budget (by avoiding overcompensation).

I begin by comparing two situations: autarky (prohibitive tariffs) and free trade. There is not necessarily free trade \textit{ex post}. Although restricted trade is possible, for simplicity, I compare autarky and free trade. The initial equilibrium is the autarky one. The uncompensated free-trade equilibrium was analyzed in the previous section. When the policymaker implements a compensation scheme, the free-trade equilibrium becomes a compensated free-trade equilibrium.

In choosing the instruments of the compensation scheme, I follow the literature in ignoring lump-sum compensation because of the associated informational requirements. Therefore, I examine a compensation scheme that is based on factor taxes and commodity taxes. (Anthony B. Atkinson and Joseph E. Stiglitz 1980, p.20) I now formally define the compensation scheme.

\textbf{Definition 2} The \textit{compensation scheme} $\sigma$ is a combination of taxes and subsidies levied on the following variables: (1) output prices, (2) generic-factor prices, and (3) occupational rewards. Tax-subsidy rates can be linear or nonlinear.

The taxes (or subsidies) on output prices are commodity taxes, and the taxes on both generic-factor prices and occupational rewards are factor taxes. Following Avinash Dixit and Victor Norman (1980,1986) and Robert C. Feenstra and Tracy R. Lewis (1994), I consider a two-stage compensation procedure. Because both Dixit and Norman (1980,1986) and Feenstra and Lewis (1994) aim to achieve Pareto-improvement, the first stage of their analysis focuses on making everyone in the economy as well off as they would be under autarky. To arrive at this end, a policymaker must utilize both commodity taxes and factor taxes—adding to these, in the case of Feenstra-Lewis, are relocation subsidies. Both Dixit-Norman and Feenstra-Lewis proved that not only will the government revenues from such first-stage schemes become non-negative, but they will be redistributed back to individuals in the economy during the second stage.

\textbf{Definition 3} The \textit{compensation scheme} $\sigma$ can be implemented in two stages:
1. In the first stage, the government tries to minimize the rents that accrue
to individual agents; in other words, it seeks to capture all these rents in the form of positive revenue. Let us call this stage’s result a $\sigma_1$ equilibrium – and

(2) In the second stage, the government sends this positive revenue back to the individual agents by means of either a poll subsidy or a reduction of some commodity taxation. Let us call the result of this second stage a $\sigma_2$ equilibrium. This $\sigma_2$ equilibrium can also be called a $\sigma$ equilibrium, since the result of the second stage is also the final result of the whole compensation scheme.

The purpose of the first stage is to ensure Pareto gains from trade by setting as close as possible to an equilibrium in which all the individual agents in the economy are as well off as they are in autarky. The first stage may leave the government non-negative revenue (or strictly positive, if there exist strict production gains from trade). The second stage tries to distribute back to individual agents the non-negative government surplus from the first-stage equilibrium. This can be done either by poll subsidy or by lowering consumption taxes (raising factor subsidies). Since the technical requirements for the second-stage redistribution – notable among these being the Weymark conditions (John A. Weymark 1979) – are closely examined in the work by Dixit and Norman (1986), I take these results as given. Our primary focus of analysis will be on the first-stage equilibrium.

First, I note the desirable and undesirable properties of the compensation scheme. Its single most important property is related to the concept of *ex post* Pareto efficiency.

**Definition 4** The compensation scheme $\sigma$ is *weakly Pareto improving* if every individual is at least as well off as he or she was under autarky.

Formally, the requirement for weak Pareto improvement is based on a comparison of individual welfare levels which, in this model, can be expressed as real income $I(\theta, \tau)$:

$$(I(\theta, \tau))^\sigma \geq (I(\theta, \tau))^A, \forall (\theta, \tau) \in \Theta,$$

where the superscript $\sigma$ denotes individual welfare under the compensation scheme, $\sigma$, and the superscript $A$ denotes individual welfare under autarky. The real income of each individual $I(\theta, \tau)$ represents the welfare level in this model, because real income equals individual indirect utility by our choice of price normalization.

Another important property of the first-stage equilibrium is that of *rent neutrality*. A positive rent arises if a policy change or change in the environment raises individual welfare. The gain is a premium or windfall profit, in the sense of a Marshallian rent. For example, if the inequality

$$(I(\theta, \tau))^\sigma > (I(\theta, \tau))^A$$

is satisfied for an agent $(\theta, \tau)$, that agent derives a strictly positive rent, with a value of $(I(\theta, \tau))^\sigma - (I(\theta, \tau))^A$, from a policy shift from autarky to free trade under the compensation scheme $\sigma$. Two-stage compensation schemes are common
in the existing literature because of the economist’s preference for discussing efficiency without addressing equity issues. Indeed, rent-neutral economic policy is desirable. That is, policy-induced arbitrary wealth redistribution, when the objective is a Pareto improvement from trade, should be avoided.

I say little about the second-stage redistribution of positive government revenues. I simply reiterate that rent neutrality is a desirable feature of the first stage of a compensation scheme. Evidence for this is that the first-stage equilibria of both Dixit and Norman and Feenstra and Lewis are consistent with rent neutrality. Next, I codify my definition of rent neutrality.

**Definition 5** The first-stage compensation equilibrium $\sigma_1$ is rent neutral if all consumers have the same utility levels as under autarky. In other words, positive rents should become government revenues.

Dixit and Norman’s original first-stage equilibrium is rent neutral. This is because all the consumers are in the same situation as they were in under autarky in the first stage. Dixit and Norman generate this result by equating both output and input prices to their levels under autarky. Fixing input prices at the autarky level guarantees autarky-level incomes for consumers. If the policymaker were to fix output prices at the autarky level, consumers would be in the same utility-maximizing situation as they were under autarky, given that only income and output prices affect the consumer’s problem. The same is true of the Feenstra and Lewis scheme. The only difference is that, in their paper, relocation subsidies are given to some consumers to compensate for the loss of income arising from positive adjustment costs associated with moving factors from one industry to another. Under the assumptions made by Feenstra and Lewis (1994), the government offers the smallest relocation subsidy consistent with some consumers being indifferent between moving and not moving to a new industry. Hence, the first-stage equilibrium in Feenstra and Lewis’s scheme is also rent neutral.

As I show subsequently, in this paper, the government cannot achieve a rent-neutral first-stage equilibrium. To achieve a Pareto improvement relative to autarky, the government must provide positive rents to some groups of individual agents. I refer to this undesirable property as overcompensation.

**Definition 6** A scheme overcompensates a group of individuals if some within that group receive positive rents in the first-stage compensation equilibrium $\sigma_1$.

Note that my definitions of overcompensation and rent neutrality represent two sides of the same coin. When the scheme is rent neutral, it does not overcompensate any group of consumers, and by same token, if the scheme is overcompensating some group, it is not rent neutral. However, I can identify the groups that receive positive rents, based on the definition of overcompensation.

The other important property of the compensation scheme concerns the budget of the government.
Definition 7 The compensation scheme, \( \sigma \), is **self-financing** if it achieves nonnegative government revenue in the first-stage equilibrium \( \sigma_1 \):

\[
B^{\sigma_1} \geq 0, \tag{28}
\]

where \( B^{\sigma_1} \) is the net government balance from the first-stage equilibrium of the scheme; i.e., the revenue from taxes minus the cost of subsidies.

This definition of a self-financing scheme is adapted from the definition of self-financing tariffs introduced by Michihiro Ohyama (1972, p. 49). A compensation scheme based on taxes and subsidies on economic variables is self-financing if the government can balance its budget solely from the net revenue earned from the scheme. The reason why equation (28) does not have a strict equality sign is that any positive revenue can be returned to individuals in the second stage.

The procedure that I adopt to design a compensation scheme is similar to those considered in Dixit and Norman (1986) and Feenstra and Lewis (1994). There are two important features. (1) The subsidies and taxes leave every consumer in the same situation as he or she was in under autarky. This policy may generate positive government revenues. (2) If there are positive revenues, the government returns these to individuals in the form of either poll subsidies or adjustments to taxes and/or subsidies. The latter is an option because, in this model, the Weymark condition of Dixit and Norman (1986) is automatically satisfied.\(^{11}\) In discussing the compensation scheme, I focus on the first step, which is to design a system of taxes and subsidies that leaves all consumers at least as well off as they were under autarky. This is because (the issues of) the second step is exhaustively discussed in the existing literature.

Another important property of any compensation scheme is its informational feasibility. Despite the fact that much of the literature (on mechanism design) discusses the concept of “feasibility” in terms of nonnegativity of governmental budgets (self-financing), this paper separates the governmental budget issues (discussed above) from the issues associated with the feasibility of a compensation scheme. In this paper, a scheme is feasible when the policy instruments of the government are based on observable variables.

Definition 8 A scheme \( \sigma \) is **informationally feasible** if it is based solely on currently observable variables. Or if it is based on the variables that are regularly considered as tax-base.

This definition of informational feasibility is based on the observability of variables by the government. (Here, the phrase observable should not be interpreted literally. The observability relates to the concept of taxability. Therefore, I claim here that the variables are observable if the policymaker can use the variable as a tax base.) What are observable variables? Which characteristics of

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\(^{11}\)The Weymark condition states that there is one good for which some consumers are net buyers and no consumer is a net seller. In traditional trade models, in which consumers are net sellers of factors of production and net buyers of consumer goods, this condition is automatically satisfied.
individuals are observable to policymakers? I propose the following three reasonable assumptions about observability. (1) The government records information on aggregate variables. (2) Therefore, it has information on aggregate variables under autarky. (3) Only current data on individuals are observed at no cost.

These assumptions make sense, because while most aggregate data is available in various forms, it is difficult to find past data that are specific to an individual. For example, the income tax rate is primarily determined by current income and does not usually depend on income from previous years. Thus, individual data for the autarky period are presumed to be costly to verify in the free-trade period.

Suppose that the government can observe (and use as tax-base) the following variables:

- $Y_1$, output prices, $P_X, P_Y$ (both at the autarky and free-trade levels);
- $Y_2$, generic-factor prices, $r$ (both at the autarky and the free-trade levels); and
- $Y_3$, residual returns (profits) from the individual’s current (free-trade) occupation.

In addition, I suppose that the government is able to observe the following two characteristics of individuals:

- $Y_4$, which industry the individual is currently working in; and
- $Y_5$, whether the individual has changed his or her occupation.

I further suppose that the government cannot observe the following variables:

- $N_1$, individual consumption vectors;
- $N_2$, individual generic-factor endowments;
- $N_3$, individual occupational-ability vectors; and
- $N_4$, residual returns (profits) from the individual’s previous (autarky) occupation.

Most of the above assumptions about observability are standard in the literature. (See, e.g., Roger Guesnerie (1995).) Given the assumption about the observability of profits, the following result can be used in subsequent analysis.

**Remark 1** Given the production set-up of the model, and given that the government can observe the residual profits of individuals, a profits tax does not distort individual behavior. In other words, individuals maximize their profits.

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12 This relates to the absence of a cumulative-profit tax system of which the late William Vickrey of Columbia University had been a proponent ever since the 1940s.
truthfully, given that the elasticity of the after-tax (subsidy) share, with respect to profit, is larger than $-1$. Formally, they do so whenever
\[ \varepsilon = \frac{\partial T/T}{\partial \pi/\pi} > -1, \] (29)
where \( T(\pi) = 1 - t(\pi) \), with \( \pi \) being residual profit and \( t(\pi) \) being an ad valorem tax rate (or, if \( t(\pi) \) is negative, a subsidy rate).

See the Appendix for a proof. Note also that the linear tax has an elasticity of \( \varepsilon = 0 \) and thus satisfies condition (29). In addition, given that individual agents are assumed to be acting truthfully, I can conclude that the policymaker can observe agent’s currently-used talents.

**Remark 2** Given the previous observation in Remark 1 about the truthfully maximized current levels of individuals’ residual returns, the government can recalculate \( \theta \) for \( X \)-producers and \( \tau \) for \( Y \)-producers. The planner can infer the amount of talent being used, as opposed to an agent’s endowment of latent talent.

This is straightforward. If policymakers can condition their policy on current profits, then either
\[
\begin{align*}
\pi_X(\theta, \tau; p) &= \left[ \frac{1}{p} \cdot (r(p))^{\frac{1}{1-a}} \cdot \left( \frac{a^\tau}{\tau} - \frac{1}{1-a} \right) \right] \cdot \theta, \text{ or} \\
\pi_Y(\theta, \tau; p) &= \left[ \frac{1}{p} \cdot (r(p))^{\frac{1}{1-a}} \cdot \left( \frac{a^\tau}{\tau} - \frac{1}{1-a} \right) \right] \cdot \tau.
\end{align*}
\]
Given the observability of aggregate variables such as the output prices, \( p \) and \( 1/p \), and the generic-factor return, \( r \), the inversion of profit to type is a simple calculation. One might also say that the profit is a strictly increasing function of the size of the type, in which case, any tax-subsidy rate that is proportional to the observed profit could be used. Hence, it is almost as if the government observes the type.

Now that I have defined all the necessary properties of the compensation scheme and examined the relevant results, I examine the results of possible compensation schemes. I investigate two distinctive cases with respect to the timing of implementation. In the first case, *an unanticipated compensation scheme*, free trade occurs before the government announces that it will compensate the losers from trade. In the second case, *an anticipated compensation scheme*, individual agents expect the compensation scheme to be implemented by the government once there is free trade. In the following section, I investigate the first case.

### 5 Two types of Compensation Schemes

Why the scheme that requires income records from two period is difficult?
5.1 Post-Trade Schemes

Surprising schemes. Despite the tradition of lump-sum compensation being introduced before trade (or the opening of the markets) (Mas-Colell, Whinston and Green 1995, p.328), a more plausible and realistic policy is a “post-trade compensation scheme” (Murray C. Kemp and Henry Y. Wan 1986, p.99), in which the government first opens up to trade, then creates the compensation scheme to help losers from trade. Arguably, this unanticipated compensation scheme was used in the 1960s. In response to the Kennedy round of GATT multilateral tariff reductions, the United States government introduced the first TAA (trade adjustment assistance) program to accommodate workers displaced by the tariff reduction.

In this section, I explore a possible unanticipated posttrade compensation policy, given the informational restrictions described. In the next section, I examine the case in which individuals anticipate the existence of the compensation scheme and analyze how this anticipation affects individual incentives.

When designing the optimal compensating redistributing scheme, it is important to consider Pareto improvements over autarky. To design such a scheme, the policymakers must be aware of the informational feasibility constraint, given the limited observability of the unused talents of individual agents. When the scheme comprises two stages, the policymakers tries to accrue all the rents in the form of governmental revenues in the first stage. Thus, the ideal first-stage equilibrium is rent neutral. Because of the informational feasibility constraint, however, my model does not posit rent neutrality of the first-stage equilibrium. Nevertheless, I explore the process of creating a compensating scheme.

For analytic convenience, I focus on the case in which the price change occurs in one direction (the other case being completely symmetric). More specifically, this is the case in which the posttrade price is $p > p^A$, and therefore, there are job switchers from sector $Y$ to sector $X$. Given the set-up of the model, described in Section 2, I consider five cases (Case I. - Case V.) relating to the gains and losses of different groups of individuals, as follows.

**Case I. Generic-factor owners all gain, since $r(p) > r(p^A)$.** Specifically, the gain for those who own $K(\theta, \tau)$ is given by

$$\left(r(p) - r(p^A)\right) \cdot K(\theta, \tau) = a \cdot K^{-1(a)} \cdot \left\{ [s(p)]^{1-a} - [s(p^A)]^{1-a} \right\} \cdot K(\theta, \tau) > 0,$$

where

$$s(p) = \frac{1}{\theta} \cdot V(\theta) + \frac{1}{\tau} \cdot V(\tau).$$

Note that this group’s gain from trade is proportional to the agent’s endowment of the generic factor, $K(\theta, \tau)$. The multiplier component,

$$a \cdot K^{-1(a)} \cdot \left\{ [s(p)]^{1-a} - [s(p^A)]^{1-a} \right\},$$

is the same for all agents. Both $a$ and $K$ are parameters of the model. Given the relative price change, $p^A \Rightarrow p$, the values for both $s(p^A)$ and
Case III. Among job-switching individuals

Case II. Job stayers in sector

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Case I, the gain from trade for job stayers in sector

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the policymaker can make the status of these individuals the same as that under autarky in the first-stage equilibrium.

Case II. Job stayers in sector X—those who are in the area \( \tau < (p^A)^{\frac{1}{1+\pi}} \theta \)—all gain, since \( \pi_{X_1}(p) > \pi_{X_0}(p^A) \) when \( p > p^A \). Specifically, the gain for those who have talent of \( \theta \) is given by

\[
\pi_{X_1}(p) - \pi_{X_0}(p^A) = K^a(1-a) \cdot \left( p^{\frac{1}{1-\pi}} [s(p)]^{-a} - p^A^{\frac{1}{1+\pi}} [s(p^A)]^{-a} \right) \cdot \theta > 0, 
\]

(33)

where the definition of \( s(p) \) is the same as in equation (31). Similarly to Case I, the gain from trade for job stayers in sector X is proportional to agents’ endowments of used talent, \( \theta \). The multiplier component,

\[
K^a(1-a) \cdot \left( p^{\frac{1}{1-\pi}} [s(p)]^{-a} - p^A^{\frac{1}{1+\pi}} [s(p^A)]^{-a} \right),
\]

is the same for all these agents. Thus, by imposing on the returns from talent of job stayers in sector X an \textit{ad valorem} tax rate of

\[
t_{\pi X} = \frac{p^{\frac{1}{1-\pi}} [s(p)]^{-a} - p^A^{\frac{1}{1+\pi}} [s(p^A)]^{-a}}{p^{\frac{1}{1-\pi}} [s(p)]^{-a}},
\]

(34)

the policymaker can make the status of these individuals the same as that under autarky in the first-stage equilibrium.

Case III. Among job-switching individuals—those who are in the area \( (p^A)^{\frac{1}{1+\pi}} \theta < \tau < (p^A)^{\frac{1}{1+\pi}} \theta \)—all gain, since \( \pi_{X_1}(p) > \pi_{Y_0}(p^A) \) when \( p > p^A \). Specifically, the gain for those who have the talent vector, \( (\theta, \tau) \), is given by

\[
\pi_{X_1}(p) - \pi_{Y_0}(p^A) = g(p^W) \cdot \theta - g(p^A) \cdot \tau > 0,
\]

(35)

where

\[
g(p^W) = p^{\frac{1}{\tau(p)}} \left( \frac{1}{r(p)} \right)^{\frac{a}{\tau}} \left( a^{\frac{1}{\tau}} - a^{\frac{1}{\pi}} \right)
\]

and where

\[
g(p^A) = p^A^{\frac{1}{1+\pi}} \left( \frac{1}{r(p^A)} \right)^{\frac{a}{\tau}} \left( a^{\frac{1}{\tau}} - a^{\frac{1}{\pi}} \right).
\]
Unlike in Cases I and II, the gain for job-switching individuals is not proportional to their endowments of used talent, $\theta$. Although $g(p^W)$ and $g(p^A)$ are the same for all these individuals and the policymaker can calculate $g(p^W)$ and $g(p^A)$, the gain, $g(p^W) \cdot \theta - g(p^A) \cdot \tau$, depends on both elements of the talent vector, $(\theta, \tau)$, which is not observed by the policymaker. The policymaker could recalculate the value of used talent, $\theta$ based on the profits from the production of $X$. However, the value of $\tau$, is not known by the policymaker. To understand this, suppose that the policymaker would like to impose an *ad valorem* tax rate of

$$t_{X-Y} = \frac{g(p^W) \cdot \theta - g(p^A) \cdot \tau}{g(p^W) \cdot \theta} = 1 - \frac{g(p^A) \cdot \tau}{g(p^W) \cdot \theta},$$

(36)

for Case III individuals. The tax rate to be imposed by the policymaker should be of the form $t_{X-Y}(\pi_X(\theta))$. That is, it should be based only on the currently observable $\pi_X(\theta)$, which depends on the current use of talent, $\theta$.

**Case IV.** Other job-switching individuals—their area is $g(p^W) \cdot \theta < \tau < p_{Y - A} \theta$—all lose since $\pi_X(1) < \pi_Y(0)$ when $p > p^A$. Specifically, the loss for those who have talent of $(\theta, \tau)$ is given by

$$- (\pi_X(1) - \pi_Y(0)) = g(p^A) \cdot \tau - g(p^W) \cdot \theta > 0.$$  

(37)

This case is quite similar to Case III, when it comes to the loss for each individual and the subsidy rate. The infeasible subsidy rate that the policymaker would like to impose on this group is

$$s_{X-Y} = \frac{g(p^A) \cdot \tau - g(p^W) \cdot \theta}{g(p^W) \cdot \theta} = \frac{g(p^A) \cdot \tau}{g(p^W) \cdot \theta} - 1,$$

(38)

whereas the feasible subsidy rate must be of the form $s_{X-Y}(\pi_X(\theta))$.

**Case V.** Job-staying individuals in sector $Y$—those who are in the area $p_{Y - A} \theta < \tau$—all lose, since $\pi_Y(1) < \pi_Y(0)$ when $p > p^A$. More specifically, the loss for those who have talent of $\tau$ is given by

$$- (\pi_Y(1) - \pi_Y(0)) = K^a(1-a) \cdot \left( p^{A - \frac{1}{q}} \left[ s(p^A) \right]^{-a} - p^{A - \frac{1}{q}} \left[ s(p) \right]^{-a} \right) \cdot \tau > 0.$$  

(39)

Similarly to Cases I and II, the gain from trade for job stayers in sector $Y$ is proportional to their endowments of used talent, $\tau$. The multiplier component,

$$K^a(1-a) \cdot \left( p^{A - \frac{1}{q}} \left[ s(p^A) \right]^{-a} - p^{A - \frac{1}{q}} \left[ s(p) \right]^{-a} \right),$$
is the same for all these agents. Thus, by imposing on the returns from
talent of job stayers in sector $Y$ an *ad valorem* subsidy rate of
\[
s_{pY} = \frac{p^{A+\tau_{Y}} [s(p^{A})]^{-a} - p^{A+\tau} [s(p)]^{-a}}{p^{A+\tau [s(p)]^{-a}}}, \tag{40}
\]
the policymaker can make the status of all the job-staying individuals in
sector $Y$ the same as it was under autarky in the first-stage equilibrium.

It is instructive to look at a first-best case, even if in reality it is impossible
to achieve. Consider the following first-best scheme.

**Case 1** As a first-stage equilibrium, tax the winning groups (Cases I, II and III)
and subsidize the losing groups (Cases IV and V) in amounts equal to their gains
and losses, so that every individual is in the same situation as he or she was
in under autarky. Such tax and subsidy rates are represented by the equations
(32), (34), (36), (38), and (40).

This hypothetical first-best scheme would be rent neutral. However, while
the taxation and subsidy schemes for Cases I, II and V are feasible, the determina-
tion of the tax and subsidy rates for the job switchers, Cases III and IV, must
be based on a combination of observable and unobservable variables. The gov-
ernment cannot distinguish between the groups in Cases III and IV because it
cannot observe the relative value of $(\theta, \tau)$ for each individual. The policymaker
can observe only the profit from current production and thus can observe, when
$p > p^{A}$, only the profit from production in sector $X$. The policymaker cannot
observe (or condition the taxation scheme on) the counterfactual profit from
sector $Y$ that is proportional to the agent’s unused latent talent, $\tau$. In terms of
Fig. 8, for instance, this means that the government cannot distinguish between
points $q$ and $r$, because in equilibrium, the individuals at these points earn the
same profit and produce the same amount of product $X$. This leads to the
following result.

**Proposition 3** Given the set-up of the model, if the government is aiming to
achieve a *Pareto improvement* over autarky, there is no *informationally
feasible* first-stage compensated equilibrium that is *rent neutral*.

By consulting the equations (30), (33), and (39), which represent the gains
and losses for the various groups of individuals, I establish the taxation and
subsidy rates for the following three groups of individuals and make them as
well off as they were under autarky: (a) owners of the generic-factor $K$, at
the rate (32); (b) job stayers in sector $X$, at the rate (34); and (c) job stayers
in sector $Y$, at the rate (40). I can do this because these individuals’ gains
and losses are proportional to their factor returns (in terms of both residual
profits and generic-factor returns), and thus also proportional to their employed
talents (or factor endowments). In this case, a linear tax or subsidy system
applies. (Recall, from Result 1 in section ??, that any linear tax-subsidy system is incentive compatible.)

I now focus on job-switching individuals. From equations (35) and (37), individual gains or loss depend on relative amounts of used talent, $\theta$ and unused latent talent, $\tau$. Because the policymaker does not have data on each individual—past profits and losses—the policymaker can base a taxation-subsidy scheme only on currently observable variables. In this case, the current profit from sector-X production is observable. In effect, the policymaker can observe $\theta$ but not $\tau$. (The policymaker observes the profits of the individual agents. If profit is reported truthfully, the policymaker can infer the amount of talent being used. See Remark 1 in section ??.) Thus, the policymaker cannot make all job-switching individuals as well off as they were under autarky, except in a case that I examine subsequently. Hence, I conclude as follows.

**Proposition 4** Given the set-up of the model, if the government is aiming to achieve a Pareto improvement over autarky, an informationally feasible posttrade compensation scheme must overcompensate job-switching individuals in its first-stage equilibrium.

If the policymaker’s most pressing concern is to ensure a Pareto improvement over autarky, then the informationally feasible scheme must overcompensate job-switching individuals. The preceding analysis shows that the policymaker can tax and subsidize job stayers in a rent-neutral manner but cannot do so for job switchers simply because the policymakers can only observe their $\theta$, not $\tau$.

I return temporarily to Fig. 8, which has a unit-square support for the joint distribution of talents. The left-hand side of the figure contains lines that represent the same percentage change in the gain or loss from trade. The right-hand side contains lines indicating that those individuals are making the same amount of residual profit. The iso-percentage-gain-or-loss lines are rays from the origin, and the iso-current-profit lines for $X$ producers are parallel vertical lines.

While this first-best scheme requires a linear taxation-subsidy system to be imposed along the iso-percentage-gain-or-loss lines, the policymaker only observes the differences between individuals along the iso-current-profit lines. This is because job-switching individuals appear the same when they are earning the same profit, and hence are represented by the same iso-current-profit line.

Of those who earn the same profit, it is individuals on the upper bound of the iso-current profit line who gain least (lose most) from trade. Since the policymaker cannot distinguish between the individuals on the same iso-profit line, the policymaker must compensate all the individuals on the same profit line at the same level as the least fortunate individual, who is on the upper bound of that line. However, apart from the least fortunate individual, individuals receiving the same amount of compensation from the policymaker have positive rents because their iso-percentage-gain-or-loss lines are higher than the individual on the upper bound.

Review the two points $q$ and $r$ in Fig. 8, which are on the same iso-current-profit line. Thus, although they appear the same to the policymaker, $q$, represents a loser while $r$ is a winner. Nevertheless, compensation must be the same.
for both. Even if the individual at \( r \) is a winner, he or she receives the same amount of subsidy (as opposed to paying a tax) as the individual at point \( q \). Hence, a government aiming for a Pareto improvement inevitably overcompensates job-switching individuals.

To explore this more thoroughly, I define the iso-current-profit set, \( I^{CP}(\theta^*) \).

**Definition 9** The iso-current-profit set, \( I^{CP}(\theta^*) \), is the set of all those job-switching individuals who have the same talent, \( \theta^* \):

\[
I^{CP}(\theta^*) \equiv \{ (\theta, \tau) \in C_{Y-X} : \theta = \theta^* \},
\]

where \( C_{Y-X} \) is a partition of job switchers; i.e.,

\[
C_{Y-X} \equiv \left\{ (\theta, \tau) \in \Theta : (p^A)^{\frac{1}{1+\tau}} \theta < \tau < p^{\frac{1}{1+\tau}} \theta \right\}.
\]

Note that \( I^{CP}(\theta^*) \) is a linear, one-dimensional subspace of \( \mathbb{R}^2 \). Let \( \tau(\theta^*) \) be the lower bound for the value of the element \( \tau \) in a set \( I^{CP}(\theta^*) \), and let \( \Theta(\theta^*) \) be the upper bound for the same subspace. Note that \( \tau(\theta^*) \) is equal to \((p^A)^{\frac{1}{1+\tau}} \theta^* \), whereas \( \Theta(\theta^*) \) depends on the value of \( \theta^* \). In particular,

\[
\Theta(\theta^*) = \sup \left\{ p^{\frac{1}{1+\tau}} \theta^*, \Theta(\theta^*) \right\},
\]

where \( \Theta(\theta^*) \) is an upper bound for the element \( \tau \) in the whole \( \Theta \) space when \( \theta = \theta^* \). In the case of a unit-square support for the joint distribution, \( \Theta(\theta^*) = 1 \).

Because all individuals in the set \( I^{CP}(\theta^*) \) are job switchers from sector \( Y \) to sector \( X \), they are currently producing output \( X \). Since all members of the set \( I^{CP}(\theta^*) \) have the same talent, \( \theta^* \), their profit is the same: \( \pi^A(p, r(p), \theta^*) \). Their individual gains or losses, however, differ because they have different latent talents, \( \tau \). Given (35) and (37), the individual gains or losses can be expressed as \(|g(p^W) \cdot \theta^* - g(p^A) \cdot \tau| \). Whether individual \( f \) (who has the talent \( \theta^* \)) gains or loses, and what the gain or loss is, depends on the value of \( \tau \). Among those who belong to the set \( I^{CP}(\theta^*) \), there are many individuals who have the latent talent \( \tau \) in the interval \([\tau(\theta^*), \Theta(\theta^*)]\). The policymaker, however, cannot distinguish between them.

A policymaker who wants to ensure Pareto gains from trade must be sure to make the least well-off individual as well off as he or she was under autarky. Note also that this least well-off individual must have had the most talent in the previous sector, \( Y \), and hence must have been the one with the most latent talent, \( \tau(\theta^*) \). Therefore, for all individuals, \((\theta^*, \tau) \in I^{CP}(\theta^*) \), the subsidy or tax must be \(|g(p^W) \cdot \theta^* - g(p^A) \cdot \tau(\theta^*)| \). The *ad valorem* rate for any individual with the profit \( \pi(\theta^*) \) is

\[
t_{\pi, Y}(\pi(\theta^*)) = \frac{g(p^W) \cdot \theta^* - g(p^A) \cdot \tau(\theta^*)}{g(p^W) \cdot \theta^*}.
\] (41)
If \( g(p^W) \cdot \theta^* - g(p^A) \cdot \tau(\theta^*) > 0 \), equation (41) represents a tax rate. If \( g(p^W) \cdot \theta^* - g(p^A) \cdot \tau(\theta^*) < 0 \), it represents a subsidy rate. With the exception of the individual at the point \((\theta^*, \tau(\theta^*))\), which represents zero, all individuals in the set \( I^{CP}(\theta^*) \) are overcompensated, since the inequality

\[
g(p^W) \cdot \theta^* - g(p^A) \cdot \tau(\theta^*) < g(p^W) \cdot \theta^* - g(p^A) \cdot \tau
\]

must hold for all those with the latent talent, \( \tau \in \left[ \tau(\theta^*), \tau(\theta^*) \right] \).

From (42), it follows that

\[
\int_{\tau(\theta^*)}^{\tau(\theta^*)} \left\{ g(p^W) \cdot \theta^* - g(p^A) \cdot \tau \right\} f(\theta^*, \tau) d\tau < \int_{\tau(\theta^*)}^{\tau(\theta^*)} \left\{ g(p^W) \cdot \theta^* - g(p^A) \cdot \tau \right\} f(\theta^*, \tau) d\tau.
\]

Integrating over all job-switching individuals yields

\[
\int_{C_Y-X} \int_{\tau(\theta^*)}^{\tau(\theta^*)} \left\{ g(p^W) \cdot \theta^* - g(p^A) \cdot \tau \right\} f(\theta, \tau) d\tau d\theta^* < \int_{C_Y-X} \int_{\tau(\theta^*)}^{\tau(\theta^*)} \left\{ g(p^W) \cdot \theta^* - g(p^A) \cdot \tau \right\} f(\theta, \tau) d\tau d\theta^*,
\]

with the integration being over \( \theta^* \) for all job-switching individuals. The difference between the right- and left-hand sides of the inequality (43) relates to the total amount of overcompensation for job-switching individuals.

These overcompensation results lead to the following proposition.

**Proposition 5** An informationally feasible posttrade compensation policy that achieves weak Pareto improvement may or may not be self-financing, depending on the joint distribution of individual talents.

According to Ohyama (1972), a Pareto-improving compensation scheme is self-financing if the set of aggregate consumption possibilities is larger than that under autarky, if there is a lump-sum transfer. In this model, however, when I impose the informational feasibility condition, a compensation scheme without a lump-sum transfer may or may not be self-financing. This is because overcompensating job-switching individuals may cancel out the positive aggregate rents arising from trade. Whether the amount of overcompensation is large depends on the shape of the joint distribution of talents. In particular, if the total mass of job-switching individuals is large, then the total amount of overcompensation is high. Some parameter values then imply that the total compensation scheme is not self-financing.

I now consider an example in which the support of the joint distribution is a unit square. Figure 9 illustrates the scheme for this case. For this unit-square case, I introduce a finer separation of the partition \( C_{Y-X} \) into two groups: a group of absolute gainers and a group of gainers and losers—based only on the observable variables. For clarification, consider the following:

(i) generic-factor owners: same as Case I;
(ii) all individuals in partition $C_{XX}$: same as Case II;

(iii) those individuals in partition $C_{YX}$ who meet the condition $\theta > \frac{g(p^A)}{g(p^W)}$;

(iv) those individuals in partition $C_{YX}$ who meet the condition $\theta < \frac{g(p^A)}{g(p^W)}$; and

(v) all individuals in partition $C_{YY}$: same as Case V.

Note that in Fig. 9, the dotted line, $OZ$, denotes the zero-gain line: $\theta = \frac{g(p^A)}{g(p^W)} \cdot \tau$. This categorization uses only observable variables because the distinction between partition (iii) and partition (iv) is based solely on $\theta$, which can be inferred from individuals’ current profits. Given this new categorization, I propose a revised posttrade compensation scheme.

**Case 2** As a first-stage equilibrium, tax (i), (ii) and (iii) and subsidize (iv) and (v). Note in particular that the tax and subsidy rates are represented by the following equations: (32) for (i); (34) for (ii); (41) for groups (iii) and (iv); and (40) for (v).

Since this scheme is based on observable variables, it is feasible. However, it is second best because groups (iii) and (iv) are overcompensated. This is inevitable given that winners and losers in this category are indistinguishable.

To find the appropriate tax-subsidy rates, I obtain the minimum subsidy rate and the maximum tax rate for each group that satisfies the weak-Pareto-improvement requirement shown in (???). Because my model uses a price normalization that ensures that nominal income is equal to real income, it is easy to find the tax-subsidy rates for all groups at which everyone is as well off as they were under autarky. Note that the tax-subsidy rates must be based on observable variables or variables that are easily calculated. Thus, the features of the tax-subsidy rates for each group are:

(i) (linear) factor (commodity) tax on generic factors;

(ii) (linear) profits tax on the occupation rewards for job-staying producers of output $X$;

(iii) (nonlinear) profits tax on the occupation rewards for job-switching producers of output $X$;

(iv) (nonlinear) profits subsidy on the occupation rewards for job-switching producers of output $X$; and

(v) (linear) profits subsidy on the occupation rewards for job-staying producers of output $Y$.

The linear factor tax for generic-factor owners is the same as that in the first-best case. Now I focus on the individual heterogeneity of talents. Given the above categorization, I denote the partitions of the ability vector space more finely, as follows.
1. \( C_{X-Y} \equiv \{ (\theta, \tau) \in \Theta : \tau < (p^A)^{\frac{1}{\tau + \epsilon}} \theta \} \)

2. \( H = C_{Y-X}^H \equiv \{ (\theta, \tau) \in \Theta : p^{\frac{1}{\tau + \epsilon}} \theta > \tau > (p^A)^{\frac{1}{\tau + \epsilon}} \theta \text{ and } 1 > \frac{g(p^W)}{g(p^A)} \cdot \theta \} \)

3. \( M = C_{Y-X}^M \equiv \{ (\theta, \tau) \in \Theta : p^{\frac{1}{\tau + \epsilon}} \theta > \tau > (p^A)^{\frac{1}{\tau + \epsilon}} \theta \text{ and } 1/(p^{\frac{1}{\tau + \epsilon}}) < \theta < \frac{g(p^A)}{g(p^W)} \} \)

4. \( L = C_{Y-X}^L \equiv \{ (\theta, \tau) \in \Theta : p^{\frac{1}{\tau + \epsilon}} \theta > \tau > (p^A)^{\frac{1}{\tau + \epsilon}} \theta \text{ and } 0 < \theta < 1/(p^{\frac{1}{\tau + \epsilon}}) \} \)

5. \( C_{Y-Y} \equiv \{ (\theta, \tau) \in \Theta : p^{\frac{1}{\tau + \epsilon}} \theta < \tau \} \)

The groups of job stayers, \( C_{X-Y} \) and \( C_{Y-Y} \), face the same linear-tax-subsidy scheme as in the first-best case. Thus, I focus on the job switchers, \( H, M \) and \( L \), all of whom are currently producing the output \( X \). Because the government cannot distinguish between those earning the same profit from their production of \( X \), the policymaker must take from (give to) each individual the same tax (subsidy) as that taken from (given to) the individual who gains the least (loses the most) among those earning the same profit. For a given profit, those who gain the least are those who have the most latent ability to produce \( Y \). For the groups \( H \) and \( M \), those who gain the least (lose the most) are the individuals with \( \tau(\theta^*) = 1 \). For group \( L \), they are \( \tau(\theta^*) = p^{\frac{1}{\tau + \epsilon}} \theta^* \).

Next, I check the optimal tax rate for those who have an ability vector \((\theta^*, 1)\), where \( 1 \geq \theta^* > 1/(p^{\frac{1}{\tau + \epsilon}}) \), and the optimal tax rate for those with a vector \((\theta^*, p^{\frac{1}{\tau + \epsilon}} \theta^*)\), where \( 0 < \theta^* < 1/(p^{\frac{1}{\tau + \epsilon}}) \). Thus, the individuals in group \( H \) who earn \( \pi(\theta^*) \) have imposed on them a tax rate of

\[
t_H(\pi(\theta^*)) = \frac{g(p^W) \cdot \theta^* - g(p^A) \cdot \theta^*}{g(p^W) \cdot \theta^* - \delta(\theta^*)},
\]

while the individuals in group \( M \) who earn \( \pi(\theta^*) \) are given a subsidy of

\[
s_M(\pi(\theta^*)) = \frac{g(p^W) - g(p^A) \cdot \theta^*}{g(p^W) \cdot \theta^*} + \delta(\theta^*),
\]

where \( \delta(\theta^*) > 0 \) represents an arbitrarily small number for which \( \delta'(\theta^*) > 0 \). The purpose of this additional term is to avoid breaching the condition \( \varepsilon = \frac{\partial T/T}{\partial \pi/\pi} > -1 \), arrived at in Result 1 of the previous section. Without this term, \( \delta(\theta^*) \), the condition is \( \varepsilon = -1 \). (For a formal proof, see the Appendix.) The group-\( L \) individuals have the linear subsidy

\[
s_L = \frac{g(p^A) \cdot p^{\frac{1}{\tau + \epsilon}} \theta^* - g(p^W) \cdot \theta^*}{g(p^W) \cdot \theta^*} - \frac{g(p^A) \cdot p^{\frac{1}{\tau + \epsilon}} - g(p^W)}{g(p^W)}.
\]

This completes the description of the tax-subsidy scheme for the first-stage equilibrium in the unit-square case.
5.2 Anticipated Schemes

In the previous section, a compensation program was implemented after the economy opened to trade. The introduction of the program is assumed to have been a surprise. This may have been the case in the 1960s, but may not describe more recent situations. Once a compensation scheme is in place, individual agents take its existence into account. They change their behavior because the program affects their incentives.\footnote{The argument is analogous to the Friedman–Phelps hypothesis about the natural rate of unemployment. Policymakers who try to take advantage of the Phillips curve by choosing higher inflation to reduce unemployment only succeed in reducing unemployment temporarily. High inflation shifts the augmented Phillips curve upwards because expected inflation at the natural rate of unemployment rises. Thus, policymakers must wait for a long time before they can take advantage of surprise inflation. By a similar logic, the policymaker cannot take advantage of an unanticipated compensation scheme for long.} In this section, I analyze an anticipated compensation scheme.

To begin, I consider the situation in which individual agents expect the compensation program to exist and they behave accordingly. In the previous section, some agents switched occupations before knowing whether there would be a compensation scheme. In this section, I posit that some individual agents who had changed their jobs under that scenario (without compensation) may not switch their occupations if they expect compensation only if they remain in a declining industry. This is inevitable, since any compensation scheme must specify the tax and subsidy rates not just for job switchers but also for job stayers. When job stayers stay in their own industry, policymakers cannot tell if they are countercfactual job switchers. Indeed, one can only tell which job stayers have changed their jobs because of the compensation scheme. With this difficulty in mind, I analyze an anticipated compensation scheme.

I use the same approach as before. In the first-stage equilibrium, the policymaker tries to make agents at least as well off as they were under autarky.\footnote{It may be necessary to provide some positive surplus for informational reasons.} Any nonnegative revenues that accrue to the government can be returned to agents in the second stage. I consider the following tax scheme for the producers of $X$ under autarky.

1. For those who stay in industry $X$, there is a linear tax rate of

$$t_{\text{ant}} = \frac{\pi_{Y0} - \pi_{X0}}{\pi_{X1}} = p^{\frac{1}{1-a}} \left[ s(p) \right]^{-\alpha} - p^{\frac{1}{1-a}} \left[ s(p^A) \right]^{-\alpha}.$$

This tax rate can make job stayers in $X$ indifferent between compensation and autarky.

2. For those who switch from industry $X$ to industry $Y$, there is a linear tax rate of

$$t_{\text{ant}} > \frac{\pi_{X1} - \pi_{Y0}}{\pi_{X1}} = p^{\frac{1}{1-a}} \left[ s(p) \right]^{-\alpha} - p^{\frac{1}{1-a}} \left[ s(p^A) \right]^{-\alpha}.$$
In practice, there are no job switchers in this direction, given the change in the terms of trade.

Thus, all members of the $C_{X^{-X}}$ group stay in industry $X$, and all must pay the amount of tax that makes them indifferent between compensation and autarky. No agent switches from $X$ to $Y$, since paying tax at the rate $t^*_{\text{aut}}$ makes no sense.

Now, to ensure that those in group $C_{Y^{-Y}}$ are at least as well off as they were under the autarky situation, I consider the following subsidy scheme for the producers of $Y$ under autarky.

3. Any producer of $Y$ under autarky who chooses to stay in industry $Y$ under free trade is granted a positive subsidy, which is proportional to his or her occupational return in producing $Y$. The linear subsidy rate is

$$s_{\text{ant}} = \frac{\pi_{Y0} - \pi_{Y1}}{\pi_{Y1}} = \frac{p^{A\pi_{Y1}} \left[ s(pA) \right]^{\alpha} - p^{A\pi_{Y0}} \left[ s(p) \right]^{\alpha}}{p^{\pi_{Y1}} \left[ s(p) \right]^{\alpha}}.$$

This offer by the government guarantees that no one is made worse off by trade, because the autarky producers of $Y$ now have the option of staying in the same industry and earning the same return as before.

The government specifies the tax-subsidy scheme for those who switch from sector $Y$ to sector $X$—namely, the group $C_{Y^{-X}}$. For a more rigorous analysis, I consider Fig. 10, in which there is a unit-square support.

I divide the unit square into five partitions. As well as natural job stayers—the groups $C_{X^{-X}}$ and $C_{Y^{-Y}}$—there are three new groups of counterfactual job switchers. These are: (1) $D$, comprising individuals who were job switchers under free trade but who remain in industry $Y$; (2) $L$, comprising winning job switchers under free trade but whose current profits are indistinguishable from those of losing job switchers; and (3) $H$, comprising winning job switchers under free trade whose current profits exceed those of losing job switchers.

With respect to group $D$, the government cannot do better than to implement the above subsidy scheme, targeting those who stay in industry $Y$. If the latter decide to stay in sector $Y$, they are indistinguishable from natural stayers in that sector. Therefore, the tax scheme targets two groups primarily: $L$ and $H$. This entails the following.

4. Tax Exemption for group $L$. Those who are in this group are natural gainers from trade. Therefore, despite the subsidy for job stayers in sector $Y$, the agents find it profitable to switch occupations, conditional on the tax exemption in the new sector.

5. Group $H$ is taxed at the same rate as in the posttrade unanticipated scheme:

$$t^*_{\text{ant}}(\pi(\theta^*)) = \frac{\pi_{X1} - \pi_{Y0} |_{\tau = 1}}{\pi_{X1}} = \frac{g(pW) \cdot \theta^* - g(pA) \cdot \theta^*}{g(pW) \cdot \theta^*} - \delta(\theta^*).$$
Then, all except those who have $\tau = 1$ gain a positive rent. Thus, this tax rate is incentive compatible for those who are in group $H$. The term $\delta(\theta^*)$ has the same property as in the previous section.

This scheme satisfies all three conditions: it has informational feasibility, it delivers weak Pareto improvement, and it is self-financing. It is informationally feasible since all tax and subsidy rates are incentive compatible. It is weakly Pareto improving since every agent is at least as well off as under autarky. If there are aggregate gains from trade, the tax revenues from this scheme exceed the costs of subsidy. The net government revenues brought in by the job-staying individuals in both sectors $X$ and $Y$ are likely to be positive. With respect to the job switchers, who created an overcompensation problem in the unanticipated case, this scheme either taxes some or exempts some from tax; hence, the policymaker generates strictly positive tax revenue. Although there are some positive rents, and hence overcompensation in the form of smaller taxes for group $H$, this overcompensation does not negatively affect the government budget since it takes the form of a smaller-than-ideal tax rate.

Nevertheless, the allocation achieved in this scheme is not without costs. Although the scheme satisfies informational feasibility, delivers weak Pareto improvement, and is self-financing, it generates aggregate-level inefficiency in the form of a smaller aggregate consumption possibility set when evaluated at the world price. Smaller aggregate gains arise because there are fewer job switchers.

**Proposition 6** There is an anticipated (ex ante) compensation program that is informationally feasible, weakly Pareto improving and self-financing. The aggregate consumption possibilities set is smaller than that of the unanticipated (ex post) scheme.

Furthermore, in the context of the current TAA program, I find a striking result. Noting that my model does not have frictional costs for occupation switching, I propose taxing at a positive rate or exempting from tax those who switch occupations. This contradicts the results in Feenstra and Lewis (1994), which propose a relocation subsidy for job switchers. My optimal scheme suggests to the contrary that the policymaker should give no subsidy to job switchers. I propose that the subsidy be given only to job stayers who remain in a declining industry. Given that the model has no frictional moving (between sectors) costs, it is no surprise to obtain this negative result relating to the current TAA, which provides a poll subsidy to occupation switchers.

**Proposition 7** The poll subsidy for those who have changed industries creates a disincentive. It induces an inefficient allocation of individuals.

Given the set-up of the model in this paper, the minimum subsidy for job-switching individuals must be nonpositive; i.e., it must contain a tax exemption for group $L$ and a positive tax for group $H$. By giving a positive subsidy to job-switching individuals, some job stayers in sector $Y$ (particularly those closer...
to the zero-gain line, $OZ$) may find it profitable to move to sector $X$. However, while this positive subsidy is successful in inducing some counterfactual job switchers to move to a more efficient sector (in the posttrade world), it also creates a huge side effect. Because the policymaker cannot distinguish between counterfactual job switchers and natural (winning) job switchers, a positive subsidy overcompensates job switchers who are on the same iso-current-profit lines. The policymaker must offer the same tax-subsidy rates that apply in the unanticipated posttrade compensation scheme if the government is maximize the number of job switchers and thereby maximize aggregate production gains. This subsidy generates overcompensation and makes self-financing questionable.

When the policymaker wants to balance the budget, taxing job switchers may be a policy option. Taxing job switchers, but not too heavily, may induce some natural job switchers to change their occupations. Since these job switchers pay tax, this policy helps to balance the budget problem but may induce fewer individuals to switch to an efficient industry. More individuals will remain in a declining industry. Thus, the trade-off between the government budget and aggregate gains remains.

The preceding analysis has shown that, in the case of an anticipated compensation scheme in which the government aims to attain a Pareto improvement over autarky, there is a trade-off between the aggregate production gains from trade and the amount of overcompensation.

6 Conclusion

The paper presented a two dimensional version of the model of occupational choice. In this paper, I have developed a model that predicts aggregate production gains from trade. I have attempted to model a realistic situation in which individual agents often find themselves. I assume that individual agents must choose one job at a time and that they are endowed with multi-valued talents in various sectors. Productivity is assumed to differ between agents. This set-up creates winners and losers from trade, but the gains and losses are based on the talents that agents use relative to their hidden latent talents. If the government chooses to impose a realistic tax-subsidy scheme on current factor prices and profits, policymakers face a trade-off between Pareto improvement and overcompensation. In other words, if policymakers do achieve a Pareto improvement, the compensation scheme necessarily overcompensates job-switching individuals. If, on the other hand, policymakers rigorously avoid overcompensation because they care about a balanced budget, the compensation program is not Pareto improving.

In addition to this trade-off, when a compensation scheme is anticipated by individual agents, there is another trade-off, which is between overcompensation and aggregate production gains. Although most policymakers are aware of these trade-offs, few studies of the issue exist. Thus, in this paper, I have developed

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15 I thank Professor Eiichi Miyagawa for pointing out the possibility of this type of policy.
a theoretical framework to explain the trade-offs that governments face when trying to implement compensating redistribution schemes.

In this paper, I have also provided an explanation of the difficulty in distinguishing winners from losers when an economy opens to trade. Such distinctions have been made in the context of basic trade models, such as the Heckscher–Ohlin model and the specific-factors model. Feenstra and Lewis (1994) noted the difficulty of identification in their imperfectly mobile factors model, which they developed to investigate heterogeneous adjustment costs. While Feenstra and Lewis assumed positive adjustment costs for their imperfectly mobile factors, my model reveals cases in which the adjustment costs for some job-switching agents may be negative and, hence, there are gainers. Thus, the poll subsidy for job-switching individuals (proposed by Feenstra and Lewis) may not be desirable in the context of my model. Furthermore, any observation of current profits does not reflect actual gains or losses from opening to trade. This makes it difficult for any government to implement a reliably Pareto-improving compensation scheme that bases taxes and subsidies on current variables.

This paper has provided a model of individuals’ occupational-choices and welfare changes when the economy faces a change in the terms of trade, and in particular, a change from autarky to free trade. I found that there are both winners and losers among job switchers. However, although this paper’s analysis can explain individuals’ long-run gains and losses from moving to a new sector, the model does not take into account short-run costs of labor adjustment. (I implicitly assumed that frictional unemployment costs are zero.) Therefore, the paper’s chief theoretical result—that no positive subsidy should be given to job-switching individuals in a self-financing compensation scheme—should not be taken too literally. Indeed, the compensation provided by the United States Department of Labor through its trade adjustment assistance (TAA) program involves a relocation subsidy for those who move to a new location when switching jobs due to trade. Such a program may be justified to the extent that there are short-run frictional costs associated with job switching.

A simplifying assumption made in this paper is that occupational talents are exogenously given for each individual. In reality, people may invest much of their time in expanding their skills. I have omitted the possibility of such dynamic development of individual talents through human-capital investment. Gene M. Grossman and Carl Shapiro (1982) analyzed the determinants of individual talent training when the individual agents are identical ex ante. An interesting extension of this paper’s model would be to incorporate a dynamic formation of specific factors, by allowing for investment in individual occupational talents. This is a promising avenue for future research.

A Appendix

This appendix collects some of the proofs.
A.1 Proof of $V_0^\prime(p) > 0$ and $V_0^\prime(p) < 0$

Here I prove it for the case of $\Theta = [0, 1]^2$:

$$\int_0^1 \int_\Theta \theta dF(\theta, \tau) = \int_0^1 \left[ \int_0^{\frac{\theta}{\theta_0}} f(\theta, \tau) d\tau \right] d\theta \equiv V_0(p)$$

Now, consider increasing the value of $p$ infinitesimally. Let us define the following

$$G(\theta) \equiv \int_0^{\frac{\theta}{\theta_0}} f(\theta, \tau) d\tau$$

and we can say that $V_0^\prime(p) > 0$ where

$$V_0^\prime(p) = \lim_{h \to 0} \frac{V_0(p + h) - V_0(p)}{h}$$

because

$$V_0(p + h) - V_0(p) = \left[ \int_0^{\frac{p+h}{\theta_0}} f(\theta, \tau) d\tau \right] - \left[ \int_0^{\frac{p}{\theta_0}} f(\theta, \tau) d\tau \right] = \int_0^1 \left[ \frac{\theta}{\theta_0} f(\theta, \tau) d\tau \right] d\theta$$

and

$$\lim_{h \to 0} \frac{V_0(p + h) - V_0(p)}{h} = \lim_{h \to 0} \frac{1}{h} \int_0^1 \left[ \frac{\theta}{\theta_0} f(\theta, \tau) d\tau \right] d\theta$$

holds. Therefore, we can conclude that

$$\lim_{h \to 0} \frac{V_0(p + h) - V_0(p)}{h} = f(\theta, \tau) \theta > 0.$$ 

$V_0^\prime(p) < 0$ can be proved in a similar manner and omitted. QED

A.2 The proof of lemma1.

From (14), we can rewrite the GNI

$$r(p) \cdot \mathbf{K} + \int_{\Theta_X} \pi_X(\cdot) dF(\theta, \tau) + \int_{\Theta_Y} \pi_Y(\cdot) dF(\theta, \tau)$$

with

$$r(p) \cdot \mathbf{K} + (r(p))^{-\alpha} \cdot \left[ a^{\frac{1}{\alpha-\alpha}} - a^{\frac{1}{\alpha}} \right] \cdot s(p)$$

and plug (13) in the form of

$$s(p) = \left( r(p) \cdot \mathbf{K}^{\frac{1}{\alpha}} / a \right)^{\frac{1}{\alpha-\alpha}}$$

and we can get (15). QED
B Profit-Tax System

Assume that the production function is

\[ x = X(k, \theta), \]  

(44)

where \( x \) is the quantity of output, \( k \) is the amount of the generic factor employed by the firm, and \( \theta \) is the specific occupational factor that is indivisible and embodied in the individual agent. Let \( X(k, \theta) \) be increasing in both arguments, strictly concave, and infinitely continuously differentiable, and let it have constant returns to scale.

Let \( p \) be the output price of \( x \). Let \( r \) be the market price for the generic factor, \( k \). The agent’s profit-maximization program is

\[ \max_k \pi(k, \theta; p, r) = p \cdot X(k, \theta) - r \cdot k. \]  

(45)

Note that the only choice variable for the agent is \( k \), because \( \theta \) is embodied and indivisible. The regular first-order condition is

\[ \frac{\partial \pi}{\partial k} = 0 \iff p \cdot \frac{\partial X}{\partial k} = r. \]  

(46)

Strict concavity of the production function, \( X(\cdot, \cdot) \), guarantees that the second-order condition for the regular problem (45) holds with strict inequality.

\[ \frac{\partial^2 \pi}{\partial k^2} < 0 \]  

(47)

Now, consider a profits tax on the profits of the agent, given equation (45). If the \textit{ad valorem} tax rate is \( t \), then the profit-maximization program is

\[ \max_k (1 - t) \{ p \cdot X(k, \theta) - r \cdot k \}. \]  

(48)

When \( t \) does not depend on \( k \) or \( \theta \), the profit-maximization problem faced by an individual is unchanged. Hence, the first-order condition is (46).

B.1 A Tax Rate Proportional to Profit

Now let \( 1 - t = T(\pi) \) be the profit-tax schedule. The rate of tax depends on the observed profit of the individual. The program is now

\[ \max_k \{ T(\pi) \cdot \pi \} = T(\pi) \{ p \cdot X(k, \theta) - r \cdot k \}. \]  

(49)

The first-order condition for (49) is

\[ \frac{\partial T}{\partial \pi} \cdot \frac{\partial \pi}{\partial k} \cdot \pi + T \cdot \frac{\partial \pi}{\partial k} = \frac{\partial T}{\partial \pi} \cdot \pi + T \cdot \left\{ \frac{\partial T}{\partial \pi} \cdot \pi + T \right\} = 0. \]  

(50)
Condition (50) implies that \( \frac{\partial \pi}{\partial k} = 0 \), except when

\[
\frac{\partial T}{\partial \pi} \cdot \pi + T = T \left(1 + \frac{\partial T}{\partial \pi} \cdot \frac{\pi}{T}\right) = T (1 + \varepsilon) = 0,
\]

with \( \varepsilon \equiv \frac{\partial T/T}{\partial \pi/\pi} \) being the elasticity of the tax rate with respect to profit. Thus, unless \( \varepsilon = -1 \), the first-order condition (50) implies the same condition as (46).

The second-order condition for the profit-maximization is

\[
\frac{\partial^2 \pi}{\partial k^2} \cdot \left\{ \frac{\partial T}{\partial \pi} \cdot \pi + T \right\} + \frac{\partial \pi}{\partial k} \cdot \frac{\partial}{\partial k} \left\{ \frac{\partial T}{\partial \pi} \cdot \pi + T \right\} \equiv SOC < 0. \tag{51}
\]

The second term of SOC is

\[
\frac{\partial \pi}{\partial k} \cdot \left\{ \frac{\partial^2 T}{\partial \pi^2} \cdot \frac{\partial T}{\partial k} \cdot \pi + \frac{\partial T}{\partial \pi} \cdot \frac{\partial}{\partial k} \right\}.
\]

This is evaluated around the optimum point, where \( \frac{\partial \pi}{\partial k} = 0 \). Thus, given (47), it follows that the relevant condition for the program’s second-order condition is

\[
\frac{\partial T}{\partial \pi} \cdot \pi + T = T (1 + \varepsilon) > 0.
\]

Given that \( T > 0 \), the condition can also be written as

\[
\varepsilon = \frac{\partial T/T}{\partial \pi/\pi} > -1. \tag{52}
\]

So, unless the profit-tax rate decreases by more than 1% as the profit simultaneously increases by 1%, the agent maximizes profit even after profit has been taxed.

### B.2 Tax Rate Proportional to Output

Now let \( 1 - t = T(x) \) be a new profit-tax schedule. The rate of tax depends on the observed output of the individual. The program is now

\[
\max_k \{ T(x) \cdot \pi \} = T(x) \{ p \cdot X(k, \theta) - r \cdot k \}. \tag{53}
\]

The first-order condition is

\[
\frac{\partial T}{\partial x} \cdot \frac{\partial X}{\partial k} \cdot \pi + T \cdot \left\{ p \cdot \frac{\partial X}{\partial k} - r \right\} = \frac{\partial X}{\partial k} \cdot \left\{ \frac{\partial T}{\partial x} \cdot \pi + pT \right\} - rT = 0. \tag{54}
\]

Note that the optimal level of \( k \) is smaller than the no-tax case (45), because

\[
\frac{\partial T}{\partial x} \cdot \frac{\partial X}{\partial k} \cdot \left\{ p \cdot X(k, \theta) - r \cdot k \right\} < 0,
\]

together with \( r > 0 \) and \( T > 0 \) implies that

\[
\left\{ p \cdot \frac{\partial X}{\partial k} - r \right\} > 0.
\]

Thus, the profit-tax system based on observed output is inevitably distortionary.
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