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Sectoral Specialisation and Growth Rate Differences Among Integrated Economies

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Sectoral Specialisation and Growth Rate Differences Among Integrated Economies*

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Abstract

This paper addresses the question of sectoral specialisation mechanisms and effects on growth rate differences providing an alternative approach to endogenous growth processes. The framework we choose draws on the Kaldorian cumulative causation approach to growth and the evolutionary modelling of technical change and industrial dynamics. The framework developed in the paper is used to consider the following issues: First, the paper addresses the question of sectoral specialisation as an emergent property of the dynamics generated by the model, focusing on the mechanisms leading to and sustaining specialisation patterns. These mechanisms are linked to technology but also demand. Second, the paper investigates the relationship between specialisation patterns and growth rate differences among economies. Specialisation can lead to increases in growth rate differences among economies. We then try to sort out the mechanisms inducing this pattern.

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1 Introduction

Understanding why growth rates differ among economies is probably the most treated issue in both empirics and theory of economic growth. It nevertheless remains an open question. The answers provided by the literature are as numerous as the theoretical approaches provided to find these answers. From the New Growth Theory to evolutionary economics, it is now widely recognised that economic growth is an endogenous process. The explanation for differences in GDP growth rates might therefore be found within these endogenous mechanisms.

We already addressed the question of the determinants of growth rate differences among economies in Llerena and Lorentz (2003). The work we present in this paper follows the tracks initiated in this previous paper. Llerena and Lorentz (2003) proposes to introduce in a unique model elements taken from Kaldorian and evolutionary literature on economic growth. From the first one we kept the idea that there exist macro-feedback mechanisms to technical change necessary to sustain growth. These are linked to demand and external trade. From the second one, we took the idea that technical change is rooted in micro-dynamics. These dynamics are uneven and stochastic by nature. This choice is driven by our conviction that these two streams of literature are complementary. On the one hand the Kaldorian approach provides a rather complete understanding of the macro-dynamics driving economic growth and its interaction with technological change. But its analysis relies on a too schematic representation of the processes underlying technical change. On the other hand evolutionary models of growth and/or industrial dynamics might respond to this schematic vision by providing detailed analysis of the micro-dynamics driving technological change[1].

This paper follows the same theoretical frame but proposes a multi-sectoral extension to the model presented in Llerena and Lorentz (2003). We then use the framework developed in this paper to consider the possible relationship between patterns of sectoral specialisation and growth rate differences among economies.

Both empirical and theoretical literature on growth recently put forward the argument that sectoral specialisation can explain patterns in growth rate differences among economies. Dalum, Laursen and Verspagen (1999), Laursen (2000) and Meliciani (2001) present empirical evidences that specialisation affects growth. Specialisation patterns are linked to the competitiveness of the economies in the various sectors. They then affect growth rate differences due to the existence of differences in the growth potential of each sectors. Some models can be found in the literature trying to reproduce these facts : Among others, Verspagen (1993), Cimoli (1994), Dosi, Fabiani, Aversi and Meacci (1994) and Los and Verspagen (2003). Verspagen (1993), Cimoli (1994) and Los and Verspagen (2003) connect specialisation patterns to the existence of structural differences in productivity dynamics among sectors and economies. The effect of specialisation patterns on GDP growth rate differences derives from demand characteristics: income elasticity (Verspagen (1993) and Cimoli (1994)) or income elasticity plus the industrial input-output structure (Los and Verspagen (2003)). Dosi, Fabiani, Aversi and Meacci (1994), develop an evolutionary

[1]We addressed this question in P.Llerena and A.Lorentz (2003) *"Alternative Theories on Economic Growth and the Co-evolution of Macro-Dynamics and Technological Change"* LEM Working Paper, Pisa

micro-founded multi-sectoral multi-country model. Authors conclude that these patterns of sectoral specialisation and GDP growth rate divergence generated by their model emerge from the interaction of micro-heterogeneity in behaviours and technological dynamics with the market selection mechanisms. Our model proposes an intermediate approach.

The paper develops a multi-sectoral growth model that links a Kaldorian macro-framework to an evolutionary modelling of technical change and industrial dynamics. The model represents the dynamics of economies linked together by external trade. Following the Kaldorian tradition economic growth is driven by the aggregate demand dynamics, derived from the balance of payment constraint. Aggregate demand dynamics is function of foreign income dynamics and of the economies relative competitiveness. The latter results from the interactions between micro-dynamics of technical change and macro-dynamics as wage dynamics. Technical change mechanisms are directly inspired by evolutionary models of growth and industrial dynamics. We aim here to replace the 'Kaldor-Verdoorn law' like process representing technical change mechanisms as traditionally found in the Post Keynesian growth literature by these micro-founded mechanisms. The micro-level of the model can be schemed as follows. Firms produce to cover consumers demand (domestic and/or external), using labour as unique production factors. Production techniques are build by firms accumulating vintages of capital; each of them characterised by its own labour productivity level. Capital vintages are developed by firms through their R&D activity. The outcome of their R&D activity is random. The resources firms can invest in these activities are constrained by their profits. Firms and therefore economies are subject to selection mechanisms through sector-wide replicator dynamics.

The next section is devoted to the presentation of the model. Section 3 reports the main results emerging from simulations and their interpretations.

2 A Cumulative Causation Growth Model with Evolutionary Micro-founded Technical Change

This section presents a multi-sectoral extension to the growth model we proposed in Llerena and Lorentz (2003). It considers economic growth as a demand-led process along Kaldorian lines. Economic growth is driven by external demand through a multiplier effect and technological change. These causal relationships are formally deduced from the balance of payment constraint.

Technical change emerges from the micro-dynamics following the evolutionary tradition. These replace the Kaldor-Verdoorn law traditionally found in the Kaldorian literature.

Macro and micro-dynamics are strongly interrelated. Aggregate demand provides the necessary resources to finance firms' technological development and therefore their competitiveness. Selection among firms and among sectors is also rooted in macro-dynamics through demand and wage setting mechanisms. Hence the macro-evolution generates the resources of the firms and the mechanisms ensuring their redistribution among the latter. In this sense the macro-frame constraints the micro-dynamics.

On the other side micro-dynamics are the core of technological change, one of the engines of growth. The competitiveness of economies relies on national firms' ability to gain productivity.

These channels constitute the circular causality between macro and micro-dynamics, driving the entire long-run growth processes.

The structure of the model can be described as follows: We consider a set of C economies integrated in an economic system through trade relations. An economy $c \in [1; C]$, is referred to with the index c. When variables are indexed w, they concern the foreign economies with regard to the economy c.

Our system counts J sectors of activity. Each economy can produce and consume products of each of these sectors. A sector $j \in [1; J]$, is referred to using the index j.

For each economy, I firms are active in each of the J sectors. A firm $i \in [1; I]$, producing in sector j and based in the economy c is referred to with the indexes i, j, c.

The entire economic system then counts C economies, J sectors, and $C*J*I$ firms. The index t refers to the time step.

2.1 The macro-economic framework: International trade, economic growth, and wage dynamics

This subsection presents the macro-economic framework of the model. The latter is decomposed in two distinct processes. First we consider GDP dynamics as deduced from the balance of payment constraint. Second, we define wage dynamics as correlated to labour productivity dynamics.

We assume that the considered economies are part of an integrated monetary system. We then excluded monetary adjustment to possible trade disequilibria. This is the case if considering as unit of analysis regions or countries in a single currency area. Economies being subject to balance of payment constraint, it thus implies that imports equal exports. Given the functional representation of imports and exports, as developed below, we can then deduce the GDP dynamics from the balance of payment constraint.

The macro-economic framework we developed here is directly rooted in the formal interpretations of Kaldor's cumulative causation approach of the economic growth process. Our formal representation found its inspiration in this respect in Thirlwall (1979) model, or in the more recent multi-sectoral models by Cimoli (1994) or Verspagen (1993), among others.

2.1.1 Balance of payment constraint and the determination of aggregate demand

For each sector j of an economy c, exports are defined as follows:

$$X_{j,c,t} = s_{j,w,t}(Y_{w,t})^{\alpha_c} z_{j,c,t} \tag{1}$$

where $Y_{w,t}$ represents the GDP of the rest of the world, computed as the sum of GDP levels of all foreign economies, $z_{j,c,t}$ represents the market share of the economy on the international markets for the sector j. α_c is the income elasticity of the rest of the world with respect to economy c exports. $s_{j,w,t}$ represents the share of income devoted to the consumption of sector j products by the rest of the world. It is formally computed as follows:

$$s_{j,w,t} = \frac{Y_{w,t}^{\varepsilon_j}}{Y_{w,t}}$$

Where ε_j represents the income elasticity of sector j products' consumption.

The market share of the economy in a sector is a proxy for the price competitiveness of the economy in the sector. It is given by the sum of the market shares of the domestic firms active in this sector:

$$z_{j,c,t} = \sum_i z_{i,j,c,t}$$

Each firm's market shares is defined through a replicator dynamic, function of firm's relative competitiveness. Hence the market share of each firm will be computed as follows:

$$z_{i,j,c,t} = z_{i,j,c,t-1}\left(1 + \phi\left(\frac{E_{i,j,c,t}}{\bar{E}_{j,t}} - 1\right)\right) \tag{2}$$

where $z_{i,j,c,t}$ represents the market share of firm i, $p_{i,j,c,t}$ the price of its product. $E_{i,j,c,t}$ stands for firm i, in sector j level of competitiveness:

$$E_{i,j,c,t} = \frac{1}{p_{i,j,c,t}}$$

$\bar{E}_{j,t}$, the average competitiveness on the international market, is computed as follows:

$$\bar{E}_{j,t} = \sum_{c,i} z_{i,j,c,t-1} E_{i,j,c,t}$$

The parameter ϕ measures the reactivity of the selection mechanism to competitiveness. Given our specification this parameter can be interpreted as a measure of price elasticity.

Imports follow the exports' specification scheme. They are function of domestic economy income, of domestic share of consumption of sector j goods, and of the rest of the world's market share. Formally imports are computed as follows:

$$M_{j,c,t} = s_{j,c,t}(Y_{c,t})^{\beta_c}(1 - z_{j,c,t}) \tag{3}$$

With

$$s_{j,c,t} = \frac{Y_{c,t}^{\varepsilon_j}}{Y_{c,t}}$$

$s_{j,c,t}$ represents the share of income devoted to the consumption of the products of sector j sector. Note that ε_j, the income elasticity of consumption of sector j products is fixed and equal across economies. The parameter β_c represents the income elasticity to import. $Y_{c,t}$ represents aggregate demand which, given the demand-led nature of the model, also defines GDP.

The growth rate of exports and imports for each sector can be deduced from these expressions as :

$$\frac{\Delta X_{j,c,t}}{X_{j,c,t-1}} = (\alpha_c + \varepsilon_j - 1)\frac{\Delta Y_{w,t}}{Y_{w,t-1}} + \phi\left(\frac{E_{j,c,t}}{\bar{E}_{j,t}} - 1\right) \quad (4)$$

$$\frac{\Delta M_{j,c,t}}{M_{j,c,t-1}} = (\beta_c + \varepsilon_j - 1)\frac{\Delta Y_{c,t}}{Y_{c,t-1}} - b_{j,c,t-1}\phi\left(\frac{E_{j,c,t}}{\bar{E}_{j,t}} - 1\right) \quad (5)$$

with

$$b_{j,c,t-1} = \frac{z_{j,c,t-1}}{1 - z_{j,c,t-1}}$$

External trades are subject to balance of payment constraint. Hence the growth rate of exports has to equal growth rate of imports. Thus:

$$\frac{\Delta X_{c,t}}{X_{c,t-1}} = \frac{\Delta M_{c,t}}{M_{c,t-1}} \quad (6)$$

With

$$X_{c,t} = \sum_j p_{j,c,t} X_{j,c,t} \text{ and } M_{c,t} = \sum_j p^m_{j,c,t} M_{j,c,t}$$

$$p_{j,c,t} = \sum_i p_{i,j,c,t}\frac{z_{i,j,c,t}}{z_{j,c,t}} \text{ and } p^m_{j,c,t} = \sum_{\bar{c}\neq c}\sum_i p_{i,j,\bar{c},t}\frac{z_{i,j,\bar{c},t}}{1 - z_{j,c,t}}$$

The balance of payment constraint can be rewritten as follows:

$$\sum_j i_{j,c,t-1}\frac{\Delta M_{j,c,t}}{M_{j,c,t-1}} + \sum_j i_{j,c,t-1}\frac{\Delta p^m_{j,c,t}}{p^m_{j,c,t-1}} = \sum_j e_{j,c,t-1}\frac{\Delta X_{j,c,t}}{X_{j,c,t-1}} + \sum_j e_{j,c,t-1}\frac{\Delta p_{j,c,t}}{p_{j,c,t-1}} \quad (7)$$

where:

$$i_{j,c,t-1} = \frac{p^m_{j,c,t-1} s_{j,c,t-1}(1 - z_{j,c,t-1})}{\sum_j p^m_{j,c,t-1} s_{j,c,t-1}(1 - z_{j,c,t-1})} \text{ and } e_{j,c,t-1} = \frac{p_{j,c,t-1} s_{w,j,t-1} z_{j,c,t-1}}{\sum_j p_{j,c,t-1} s_{w,j,t-1} z_{j,c,t-1}}$$

$i_{j,c,t-1}$ and $e_{j,c,t-1}$ weigh the importance of each sector' s dynamics in gross imports and exports dynamics. These two components reflect the sectoral structure of the economy. Their changes through time illustrate the structural changes in the economies.

The introduction of the balance of payment constraint allows us to express the GDP growth rate as function of the rest of the world GDP growth rate and of market share growth rate. Formally GDP growth rate will be computed as follows:

$$\frac{\Delta Y_{c,t}}{Y_{c,t-1}} = \gamma_{c,t-1}\frac{\Delta Y_{w,t}}{Y_{w,t-1}} + \lambda_{c,t-1}\phi\left[\sum_j \theta_{j,c,t-1}\left(\frac{E_{j,c,t}}{\bar{E}_{j,t}} - 1\right)\right] + \lambda_{c,t-1}\sum_j \kappa_{j,c,t-1} \quad (8)$$

With

$$\gamma_{c,t-1} = \frac{\alpha_c + \sum_j e_{j,c,t-1}\varepsilon_j - 1}{\beta + \sum_j i_{j,c,t-1}\varepsilon_j - 1} \quad (9)$$

$$\lambda_{c,t-1} = \frac{1}{\beta + \sum_j i_{j,c,t-1}\varepsilon_j - 1} \tag{10}$$

$$\theta_{j,c,t-1} = e_{j,c,t-1} + i_{j,c,t-1} b_{j,c,t-1} \tag{11}$$

$$\kappa_{j,c,t-1} = e_{j,c,t-1}\frac{\Delta p_{j,c,t}}{p_{j,c,t-1}} - i_{j,c,t-1}\frac{\Delta p^m_{j,c,t}}{p^m_{j,c,t-1}} \tag{12}$$

The first component of the right hand side of the equation captures a trade multiplier like effect on GDP growth rates. The second and third components mirror the effects of technological change on GDP dynamics through respectively the linkage between sectoral competitiveness and GDP growth, and between price changes and GDP growth. This representation allows a clear decomposition between the effect of external demand and of technological change on the 'short-run' GDP dynamics. The relative weight of these components is strongly linked to the structural characteristics of the economy. These characteristics are themselves subject to changes along time, due to the evolution of demand and technological change, leading to more complex interactions in defining the long-run growth patterns than in this short-run specification.

2.1.2 Wage determination:

Wages are set at the sectoral level. For a given sector j wage dynamics will be correlated to sector j productivity growth rate ($\frac{\Delta A_{j,c,t}}{A_{j,c,t-1}}$) and to the entire economy productivity growth rate ($\frac{\Delta A_{c,t}}{A_{c,t-1}}$). The effect of these two variables on wage dynamics is weighted by the parameter $\nu \in [0;1]$, such that :

- When $\nu = 1$, the wage dynamics for every sector only depend on the macro-level productivity growth rate. (i.e. as a centralised wage negotiation system)
- When $\nu = 0$, the wage dynamics for every sector only depend on the sector-level productivity growth rate. (i.e. as a sectoral wage negotiation system)

Wage dynamics of the sector j, in the economy c is represented as follows:

$$\frac{\Delta w_{j,c,t}}{w_{j,c,t-1}} = \nu\frac{\Delta A_{c,t}}{A_{c,t-1}} + (1-\nu)\frac{\Delta A_{j,c,t}}{A_{j,c,t-1}} \tag{13}$$

With

$$A_{c,t} = \frac{Y_{c,t}}{L_{c,t}} \text{ and } A_{j,c,t} = \frac{Y_{j,c,t}}{L_{j,c,t}}$$

Note that the wage level defined with this process during the period t is applied by firms at period $t+1$. Wage dynamics in our model act as a second macro-constraint on firms. Hence, it affects directly firms competitiveness and then the effect of the selection mechanisms on firms. Firms in a given sector of an economy will loose competitiveness if their own productivity growth rate is slower then the average one. Moreover, when $\nu \neq 0$, wage dynamics generate a selection process among sectors. Hence, if the average productivity of a sector grows slower then the average productivity growth rate of the entire economy, through wage dynamics, this sector looses competitiveness. The amplitude of this effect directly depends on the value of the parameter ν. As argued in the last section of this paper wage dynamics through the process described above play a major role in the specialisation dynamics.

2.2 Firms: production, construction of production capacity

This subsection is devoted to the description of the microeconomic level of the model. We consider here the formal representation of firms' production capacities, investment decisions and R&D activity. Note that the representation provided is common to all sectors and economies. Sectoral or economy-wide specificity, when considered, takes the form of specific parameter values.

Following the evolutionary tradition we consider a population of bounded rational firms that can differ in their technological characteristics (i.e. productivity level and dynamics) and behaviours. Technical change emerges at the firm level as a mutation process. More precisely technical change is embodied in capital vintages developed by firms to build and improve their production capacities. In this respect the model is close to Silverberg and Verspagen (1995), even if diverging in the formal representation of technical change.

Firms then play two specific role in the model. First they satisfy the demand needs. This provides them with the necessary resources to sustain the development of their production capacities. Second, through this process they generate technical change. The latter then affects the macro-dynamics, increasing the economy competitiveness and therefore affects demand dynamics.

2.2.1 Production and pricing

Firms' production process is represented by a Leontiev production function with labour as unique production factor. Capital goods enter the production function in defining labour productivity. The production function is represented as follows :

$$Y_{i,j,c,t} = A_{i,j,c,t-1} L^{p}_{i,j,c,t} \tag{14}$$

where $Y_{i,j,c,t}$ is the output of firm i, producing in sector j at time t. $A_{i,j,c,t-1}$ represents labour productivity and $L^{p}_{i,j,c,t}$ the labour force employed in the production process. Output is constrained by the demand directed to the firms and defined at the macro-economic level. The level of production of each firm is computed as a share of sector j demand[2] given by their relative market share such as:

$$Y_{i,j,c,t} = \frac{z_{i,j,c,t}}{z_{j,c,t}} Y_{j,c,t}$$

Labour productivity is function of the firms' accumulation of capital goods. Each capital good embodies a level of labour productivity. Investment in the different vintages of capital goods modifies the labour productivity of the firm. Hence, at the end of any period t, we define the level of labour productivity as follows:

$$A_{i,j,c,t} = \frac{I_{i,j,c,t} a_{i,j,c,t-1}}{\sum_{\tau=1}^{t} I_{i,j,c,\tau}} + \frac{\sum_{\tau=1}^{t-1} I_{i,j,c,\tau}}{\sum_{\tau=1}^{t} I_{i,j,c,\tau}} A_{i,j,c,t-1} \tag{15}$$

where $a_{i,j,c,t-1}$ represent the labour productivity embodied in the capital good developed by the firm i during the period $t-1$. $I_{i,j,c,t}$ represents the level of investment in capital goods of the firm.

[2]Sector j demand is computed as: $Y_{j,c,t} = X_{j,c,t} + s_{j,c,t}(1 - (Y_{c,t})^{\beta_c - 1}) Y_{c,t} z_{j,c,t}$.

Firms set prices through a mark-up process. This mark-up is applied to unitary production costs, corresponding here to labour costs. Prices are computed as follows:

$$p_{i,j,c,t} = (1+\mu_j)\,\frac{w_{c,t-1}}{A_{i,j,c,t-1}} \tag{16}$$

where $p_{i,j,c,t}$ represents the price set by firm i at time t, μ_j the mark-up coefficient and $w_{j,t-1}$ the wage level set at the macro level for the entire sector. Note that we assume here that the mark-up coefficients are fixed for each firm in a given sector of a given economy.

Firm's profit level will then be computed as follows:

$$\Pi_{i,j,c,t} = p_{i,j,c,t}Y_{i,j,c,t} - w_{c,t-1}L^p_{i,j,t} = \mu_j \frac{w_{c,t-1}}{A_{i,j,c,t-1}} Y_{i,j,c,t} \tag{17}$$

Profits constitute in the model the only financial resource for firms' investments.

2.2.2 Building production capacities

As introduced previously, to build but also improve their production capacities, firms have to accumulate capital vintages. Each capital good is developed in-house by firms and then introduced in their production technologies. This process is decomposed in two phases. First firms explore and develop new capital goods, through local search or through the adaptation of existing capital goods to their own production techniques. This phase takes place within the R&D activity of the firms. The latter is financed by investments in R&D. The second stage consists in introducing the outcome of the R&D activity within the production process. This stage is costly and requires firms to invest in the exploitation of the latest capital good vintage. The level of investment determines the relative importance of the latest capital goods in the production process and therefore determines the effective productivity gains, as described above. These two distinct investments are subject to the firms financial constraint. Firms' only resources for investments are their profits. More profitable firms are more inclined to invest and therefore to improve their production capacities and their competitiveness.

The investment decision timing is set as follows, first firms invest in capital goods, in order to gain from the already developed vintages, and then invest in R&D. Investment in capital goods corresponds to a share $\iota_{i,j,c}$ of firms' sales. Given the financial constraint the investment level in capital good is formally represented as follows:

$$I_{i,j,c,t} = min\left\{\iota_{i,j,c}Y_{i,j,c,t}\ ;\ \Pi_{i,j,c,t}\right\} \tag{18}$$

Investments in R&D are a share $\rho_{i,j,c}$ of their sales. R&D investment will correspond to the hiring of workers assigned to the research activity :

$$R_{i,j,c,t} = \frac{1}{w_{c,t-1}} \min\{\rho_{i,j,c}Y_{i,j,c,t}; \Pi_{i,j,c,t} - I_{i,j,c,t}\} \tag{19}$$

The formal representation of the R&D process is explicitly inspired by evolutionary modelling of technical change. Hence following Nelson and Winter

(1982) we will consider that the probability of success of research is an increasing function of R&D investments. Formally the R&D activity is represented by the following algorithm:

1. Firms draw a number from a Uniform distribution on [0 ; 1].
2. If this number is contained in the interval $[0\ ;\ \frac{R_{i,j,c,t}}{Y_{i,j,c,t}}]$, the R&D is successful. Hence a new capital good vintage has been developed.
3. If R&D is successful, its outcome is drawn from the following distribution. We differentiate here explicitly innovative firms from imitative ones:

$$a_{i,j,c,t} = \max\{a_{i,j,c,t-1} + \epsilon_{i,j,c,t}; a_{i,j,c,t-1}\} \tag{20}$$

$$\epsilon_{i,j,c,t} \sim N(0; \sigma_{i,j,c,t}) \tag{21}$$

$$\text{with} \begin{cases} \sigma_{i,j,c,t} = \sigma_{j,c} & \text{if the firm is an innovator} \\ \sigma_{i,j,c,t} = \chi_{j,c}(\bar{a}_{j,t} - a_{i,j,c,t}) & \text{if the firm is an imitator} \end{cases} \tag{22}$$

The outcome of the R&D process defines the labour productivity level embodied in the newly discovered capital vintage ($a_{i,j,c,t}$). $\bar{a}_{j,t}$ represents the average productivity level embodied in the latest capital vintages developed by firms. It is formally computed as:

$$\bar{a}_{j,t} = \sum_{i,c} z_{i,j,c,t} a_{i,j,c,t-1}$$

Hence $\bar{a}_{j,t} - a_{i,j,c,t}$ represents firm i, j, c technological gap, while the parameter $\chi_{j,c} \in [0; 1]$ can be seen as the degree of access to spillover for the imitating firms.

Firms exit the market if their market share is lower then $\bar{z_j}$. They are replaced by firms with a productivity level and value of the latest capital vintage developed equal to the average values of these variables within the sector and economy of the exiting firms and a market share equal to $\bar{z_j}$. In this respect the number of firms remains constant. An exiting innovator is replaced by an entrant innovator, and an exiting imitator by an entrant imitator. The proportion of innovators, and thus imitators, then remains constant.

3 Sectoral Specialisation and Growth Rate Differences: Some Simulation Results

The model, as detailed in the previous section, is developed to consider the determinants of sectoral specialisation and their effect on growth rate differences among the integrated economies. We do not assume here any ad-hoc specialisation. We rather look for specialisation to emerge from the dynamics generated through the model.

Some models can be found in the literature that raise the question of the emergence of specialisation patterns. Due to some similarities on the theoretical ground, one might particularly think about Verspagen (1993), Aversi, Dosi, Fabiani and Meacci (1994), Cimoli (1994) or Los and Verspagen (2003) among others. Verspagen (1993), Cimoli (1994) and Los and Verspagen (2003) connect specialisation patterns to the structural differences among economies in the sources of technical change and productivity gain. They base their analysis on the Kaldor-Verdoorn law. These models then link specialisation patterns to GDP growth rate differences through the differences in income elasticity of sectors' demand. In this respect these models are close to the Kaldorian analysis of growth rate differences. Dosi, Fabiani, Aversi and Meacci (1994), develop an evolutionary micro-founded multi-sectoral growth model. They show the emergence of significant patterns of GDP growth rate divergence among economies. These can be coupled to sectoral specialisation patterns in some economies. Authors conclude that these patterns emerge from the interaction of micro-heterogeneity in behaviours and technological dynamics with the market selection mechanisms. They thus consider these patterns as micro-driven.

We consider here economies initially identical, endowed with the same potentials of productivity growth, and equal access to spillovers. Technological heterogeneity among firms and therefore economies in our model results from the stochastic generation of technical change at the micro-level.

As for most of the models incorporating evolutionary features, we need to resort to numerical simulations.[3] Simulations are set through the following scheme. Our artificial system counts 4 economies and 5 industrial sectors. Each economy is producing and consuming the output of each of these sectors and counts 20 active firms per sectors. An economy is then composed of 100 firms, and each sector counts 80 firms competing against each others. In each sector, and each country half of the firms are set being innovators and therefore half of them are imitators. Exiting innovators are replaced by entering innovators, so that this proportion remains constant. All firms and all economies are initially similar, in terms of initial conditions and parameter settings.[4]
Our analysis focuses on the effect of two groups of parameters. A first one concerns macro-components of the model, while the second considers technological parameters. The macro-level parameters are the following:

- ν, the parameter weighting the effect of sector versus aggregate productivity growth rates in the sector-level wage dynamics. This parameter generates a selection among sectors, favouring the most dynamic ones in terms of productivity increases, through the relationship between wage and prices and therefore competitiveness.

- ϕ, the price elasticity, included in the replicator equation, directly influences the speed of the selection process among firms in a given sector. This parameter should somehow regulate the amplitude of the specialisation process.

- ε_j, for which we consider the effect of growing inter-sector heterogeneity. Income elasticity differences are usually considered in the literature as a source of GDP growth rate differences when specialisation occurs. Economies specialising in higher elasticity sectors (i.e. with a high demand potential) should grow faster.

[3]Simulations are implemented using the Laboratory for Simulation Development (LSD) environment. See Valente and Andersen (2002)

[4]The details of the parameter values can be found in appendix.

The set of technological parameters is the following:

- $\sigma_{j,c}$ is a parameter of the stochastic process defining the outcome of a successful R&D activity for innovators. It can be interpreted as the range of technological opportunities. We consider here the effect of a growing heterogeneity of technological opportunities among sectors. By doing this we impose some structural differences among sectors in their potential of productivity gains.

- $\chi_{j,c}$ defines the appropriability of technological spillovers. It influences imitators ability to access and adopt more advanced technologies, and then reduce their technological gap. A greater appropriability of spillovers should therefore limit productivity differences among economies in a given sector.

The next subsections are devoted to the description and the interpretation of some of the simulation results. For each parameter configurations, the results presented reflect the average value of the considered variables over 20 simulations. Each simulation lasts 500 steps.

3.1 Some patterns of sectoral specialisation and their determinants

Our first concern is to investigate the factors influencing specialisation patterns emerging from the dynamics generated by the model.

The level of specialisation is measured through the inverse Herfindahl index of sectors' production shares. Note that we do not differentiate between the share of production for the foreign and the domestic markets. We compute this index as follows:

$$H_{c,t} = \left[\sum_{j=1}^{5} \left(\frac{p_{j,c,t} Y_{j,c,t}}{\sum_{j=1}^{5} p_{j,c,t} Y_{j,c,t}} \right)^2 \right]^{-1}$$

This index estimates the number of sectors in which production is concentrated. Given the specification of our model, this indicator is defined in the interval $[1;5]$. When $H_{c,t}$ equals 5, the economy c produces the same level of output along the 5 sectors. In other words, the economy do not specialise its production in a specific sector. When $H_{c,t}$ equals 1, the production of the economy c is concentrated in a specific sector. It is then highly specialised.

Our analysis of specialisation concentrates on the average specialisation level among the 4 economies composing the system. We refer along this section to $\bar{H}_t$ that is computed as follows:

$$\bar{H}_t = \frac{1}{4} \sum_{c=1}^{4} H_{c,t}$$

Figures 1 to 4 report the average specialisation level $\bar{H}_t$ after 500 simulation steps for a selected set of parameter settings. The parameter configurations considered here aim to underline the importance of the macro-frame on the economy dynamics by catalysing, amplifying or absorbing the effects of the technological micro-dynamics. We therefore choose to confront settings of the parameter ν, controlling for inter-sector selection mechanisms to increases in price elasticity ϕ (Figure 1), increases in heterogeneity in income elasticity (Figure 2), increases in technological opportunity (σ_j) heterogeneity (Figure 3) and increases in the absorptivity of spillovers (χ) (Figure 4).

The results presented in Figure 1 confirm our intuition on the role of catalyser played by the wage setting mechanisms. Hence, whatever the parametrisation, as long as $\nu \neq 0$, not only specialisation occurs but its level increases (i.e. $\bar{H}_t$ decreases), as ν increases (see Figure 1 and 5). These results are directly linked to the cumulative nature of productivity gains (through investments in capital goods) combined with the stochastic nature of technical change generates and reinforces productivity gaps among firms and then potentially among sectors. Hence as presented in Figure 7, the model generates significant differences in productivity growth rates among sectors, even if initially equal. With $\nu > 0$, through wage dynamics and its effect on competitiveness, it magnifies the heterogeneity among sectors. These productivity gaps among firms and sectors then undeniably lead to sectoral specialisation. These differences are amplified when the inter-sector selection mechanisms are active and increased (Figure 5) when increasing ν. In other words by fostering selection between sectors, wage dynamics influences directly the productivity dynamics, fostering specialisation. Specialisation is therefore itself a cumulative and self-reinforcing process. Hence For small values of ν, these mechanisms are significantly amplified by increases in price elasticity (ϕ), as depicted in Figure 1 and 6.

A second significant specialisation pattern is to be found when increasing differences in income elasticity among sectors, as shown by Figure 2. Heterogeneity in income elasticity leads to sectoral specialisation. In this case, specialisation is not only driven by technological dynamics but also by the structure of aggregate demand and it evolution. Hence, even for $\nu = 0$ (Figure 8), growing the heterogeneity in ε_j generates patterns of concentration of production in a limited number of sectors. The specialisation level grows with the heterogeneity. The mechanisms described in the previous case (when growing ν) are here neutralised. Specialisation is therefore deterministically led by demand. Moreover, as depicted in the first picture in Figure 2 and 8, differences in income elasticity seem to affect significantly both the speed and the range of sectoral specialisation.

This process seems however annihilated for high values of ν (Figure 2). For low values of ν, the demand effect dominates the effect linked to technical change. It seems however to gradually disappear while increasing ν as shown by Figure 2. The mechanisms linked to the selection mechanisms then dominates.

The next two considered parameters concern the micro-level technological characteristics. More precisely we investigate here the effect of increasing differences in technological opportunities (σ_j) among sectors (Figure 3) and of growing the appropriability of technological spillovers (χ_j) (Figure 4). These parameters influence directly the processes generating technical change. If the effect led by ν is directly linked to the fact that technical change can unevenly occurs among firms and sectors, it is therefore highly expected that these parameters also influence specialisation patterns:

First, the changes in σ_j might therefore reinforce productivity gaps among sectors by providing significantly different technological opportunities. Figure

3 seems nevertheless to contradict this intuition. Hence for any $\nu \neq 0$, when the differences in technological opportunities grows, the specialisation level decreases. For the highest level of heterogeneity, $\bar{H}_{t=500}$ take values around 2. On average, along the 4 economies, production is therefore concentrated in 2 sectors. This result might be explained as follows: With high differences in technological opportunities, economies concentrate their production in the most dynamic sector. The remaining 4 sectors require demand to be satisfied[5]. Economies might therefore specialise in a second sector of activity. This possible explanation is sustained by the results presented in Figure 11. When considering highly heterogenous technological opportunities between sectors, the production tends to concentrate on average on the most favoured sector, while the rest of the production is distributed among the remaining ones. This process might then take place due to productivity gaps among the remaining sectors. In other words, with high heterogeneity in technological opportunities we might observe a second order specialisation process.

Second, increasing χ_j is supposed to reduce technological gaps among firms in the same sector. Therefore, if productivity gaps emerge among economies, the latter should tend to reduce through imitation with high values of χ_j. Imitation should not affect productivity differences among sectors. Growing χ should therefore contribute to maintain or even increase differences among sectors and thus affect specialisation. As depicted by Figure 4, increasing the value of χ_j does not significantly affect specialisation.

To briefly summarise the results detailed in this section, one might note the predominance of two main specialisation regimes :

- Uneven technical change among firms and sectors, reinforced by the cumulative nature of productivity gains is one of the major forces driving specialisation. It however requires some diffusion channels across sectors. This role is played by wage dynamics. Moreover, this might predominate over all the other mechanisms.

- The sectoral concentration of production can also be demand-led. This effect is directly linked to the demand-driven nature of our model. Demand constrains production. Therefore economies might natural concentrate their production toward sectors with the highest income elasticity.

These two regimes are complementary in explaining the emergence of specialisation patterns. Hence specialisation can be driven by technology, by demand or the both. In any case the macro-frame plays a determinant role, first in catalysing technology dynamics and diffusing them at the macro-level, and second the macro-structure of demand can have a direct influence on the specialisation patterns.

3.2 Sectoral specialisation and GDP growth rate differences

Our principal concern when considering patterns of specialisation is their possible connection with patterns of GDP growth rate differences. This subsection

[5] Note that this result occurs only when sectors are characterised by equal income elasticities.

proposes to present and interpret the outcome of simulations in terms of growth rate differences. We resort to the same parameter settings as for specialisation. These differences are measured through the coefficient of variation in GDP growth rates among the 4 economies over 500 simulation steps. We recall that the coefficient of variation is defined as the ratio between standard deviation and absolute average. This indicator provides a measure of relative variability.

Figures 11, 13, 15 and 16 present the average coefficient of variation in GDP growth rates among economies along the 500 simulation steps for the various parameter configuration, and Figures 12 and 14 report the dynamics of this indicator along the 500 simulation steps for some parameter specifications. A first look at the results tends to sustain the idea that specialisation patterns and growth rate differences patterns are connected. Hence parameter settings leading to significant specialisation patterns also lead to increases in the GDP growth rate differences among economies.

Figure 11 depicts the effect of increasing selection parameters: ϕ and ν. As for specialisation, increasing ν for given values of ϕ generates growing differences in GDP growth rates. In this case, these differences are triggered by the micro-dynamics of technical change. Wage dynamics is the channel allowing micro-processes to affect these macro-patterns. The model reproduces here the causal relations to be found in Dosi, Fabiani, Aversi and Meacci, driving growth rates differences. Nevertheless, when considered in absolute terms, growth rates differences generated by these mechanisms remain quite low. When considering the dynamics of the growth rate differences (Figure 12), simulation clearly exhibits that the differences in growth concentrates around the first 100 time periods; differences gradually fade and become marginal in the last 100 steps. In this case, differences in GDP growth rates are directly linked to differences in productivity levels. There transitory nature might be explained by the specialisation process. When specialised, economies have quasi monopolist positions in the sector there specialised in, technology dynamics affect growth through changes in competitiveness that have no more effect on growth in case of monopoly.

Figures 13 and 14 present the patterns of growth rates differences emerging when increasing the heterogeneity of income elasticity among sectors. The differences in growth are explained by differences in demand characteristics. The income elasticity differences then generate high and low growth path. Specialisation led by technical change then pushes economies on the tracks of one or the other. In this case the model generates growth rate differences patterns in line with the Kaldorian argument, without assuming structural differences in productivity gains. Specialisation and growth differences patterns emerge from the co-evolution of aggregate demand and micro-based technical change. Contrary to the previous parameter configurations, when considering heterogenous income elasticities, the differences in growth rates are not only transitory. The differences remain significant over time as depicted in Figure 14.

Figure 15 report the patterns of growth rate differences emerging when increasing the heterogeneity in technological opportunities. In this case again, the increasing heterogeneity leads to larger differences in GDP growth rates. Note that in this case the effect is particularly significant when coupled to high values of ν. Technological differences require the inter-sector selection mechanisms provided by wage dynamics to affect growth rate differences. Note also that for the same reasons than exposed above, these differences in growth rates are only transitory and also fade with the specialisation dynamics leading to sectoral monopolies.

As for specialisation, growing the appropriability of technological spillovers do not exhibit clear patterns in growth rate differences (Figure 16).

To summarise the results provided by simulations, we might first stress that the main drivers for specialisation, also generates growth rates differences among economies. Hence, specialisation emerges from the heterogeneity in technical change generated by the micro-dynamics. The latter are amplified by the inter-sector selection process provided at the macro-level by the wage dynamics. These mechanisms also generate growing differences in GDP growth rates. But these differences are concentrated around the first periods and fades while the specialisation process leads to sectoral monopolies.

Second, demand factors also influence the concentration of production in a limited number of sectors, this even when neutralising the effect of technical change at the same time as inter-sector selection. These demand factors as represented by heterogeneous income elasticity also exert a major effect on patterns of growth rates differences. Contrary to the previous cases, differences in GDP growth rates are permanent.

Hence factors leading to specialisation also generate significant differences in GDP growth rates. Two regimes emerge from the simulation, the first is linked to technology dynamics and selection mechanisms and the second is linked to the evolution of the demand structure and demand characteristics. If both generate growth rate differences, in the first regime these are only transitory while in the second they are permanent. This confirms and completes the results found in Llerena and Lorentz (2003) with a one-sector model.

4 Concluding Remarks

This paper attempts to pursue the analysis of the determinants of growth rates differences among economies we started in Llerena and Lorentz (2003). In this paper we developed a model of cumulative causation growth on Kaldorian lines replacing the Kaldor-Verdoorn law by evolutionary micro-founded mechanisms for technical change.

We propose here a multi-sectoral extension of this model. With this new framework, we consider another dimension in the possible determinants of growth rate differences to be found in the literature: Sectoral specialisation. In both empirical and theoretical literature, a growing number of contributions stresses the importance of patterns of specialisation in explaining these differences. This relationship can be linked to sectoral differences in technological factors, demand factors or both at the same time.

We resort to numerical simulations to address this issue using the framework developed in the second section of this paper. Our investigations focus on the effect of a selected number of parameters on specialisation and growth rate differences patterns. Among these parameters, two are related to the demand factor : price and income elasticity. A third one regulates wage dynamics. The remaining two are linked to technical change.

The results provided by simulation tend to be in line with other existing models. These results are only preliminary and require to be confirmed by a deeper analysis of the model. Still they provide already a few interesting insights.

The main drivers for specialisation, also generates growth rates differences among economies. Specialisation emerges from the differences in productivity gains generated by the micro-dynamics, through an inter-sector selection channel provided by wage dynamics. In our case, the sources of productivity grow are not assumed to structurally differ among sectors, as in Verspagen (1993), Cimoli (1994) or Los and Verspagen (2003). In this sense our model shows that we do not necessary have to assume these structural differences to observe specialisation patterns.

Simulations also emphasise the influence of demand factors on patterns of concentration of production in a limited number of sectors, this even when neutralising the effect of technical change at the same time as inter-sector selection. The influence of the demand structure coupled with the undeniable catalyser mechanisms played by wage dynamics also stress the importance of the macro-frame in diffusing specialisation and growth impulses from micro to macro-dynamics.

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A Simulation settings

	Sector 1	Sector 2	Sector 3	Sector 4	Sector 5
ν	0	0	0	0	0
ν	0.05	0.05	0.05	0.05	0.05
ν	0.1	0.1	0.1	0.1	0.1
ν	0.15	0.15	0.15	0.15	0.15
ν	0.2	0.2	0.2	0.2	0.2
ν	0.25	0.25	0.25	0.25	0.25
ν	0.5	0.5	0.5	0.5	0.5
ν	0.75	0.75	0.75	0.75	0.75
ν	1	1	1	1	1
ϕ	0.25	0.25	0.25	0.25	0.25
ϕ	0.5	0.5	0.5	0.5	0.5
ϕ	0.75	0.75	0.75	0.75	0.75
ϕ	*1*	*1*	*1*	*1*	*1*
ϕ	1.25	1.25	1.25	1.25	1.25
ε_j	*0.2*	*0.2*	*0.2*	*0.2*	*0.2*
ε_j	0.175	0.225	0.2	0.2	0.2
ε_j	0.15	0.25	0.2	0.2	0.2
ε_j	0.125	0.275	0.2	0.2	0.2
ε_j	0.1	0.3	0.2	0.2	0.2
σ_j	*0.1*	*0.1*	*0.1*	*0.1*	*0.1*
σ_j	0.075	0.125	0.1	0.1	0.1
σ_j	0.05	0.15	0.1	0.1	0.1
σ_j	0.025	0.175	0.1	0.1	0.1
χ_j	0	0	0	0	0
χ_j	0.25	0.25	0.25	0.25	0.25
χ_j	0.5	0.5	0.5	0.5	0.5
χ_j	*0.75*	*0.75*	*0.75*	*0.75*	*0.75*
χ_j	0.1	0.3	0.2	0.2	0.2

Table 1: Key parameters settings (the values by default are in italic)

	Economy 1	Economy 2	Economy 3	Economy 4	
α_c	0.45	0.45	0.45	0.45	
β_c	0.4	0.4	0.4	0.4	
	Sector 1	Sector 2	Sector 3	Sector 4	Sector 5
μ_j	0.6	0.6	0.6	0.6	0.6
$\bar{z}_j$	0.001	0.001	0.001	0.001	0.001
$\iota_{i,j,c}$	0.4	0.4	0.4	0.4	0.4
$\rho_{i,j,c}$	0.2	0.2	0.2	0.2	0.2

Table 2: Other parameters (set equally among economies, sectors and firms)

	Economy 1	Economy 2	Economy 3	Economy 4	
$Y_{c,t-1}$	100	100	100	100	
$Y_{w,t-1}$	301	301	301	301	
$z_{j,t-1}$	0.25	0.25	0.25	0.25	
	Sector 1	Sector 2	Sector 3	Sector 4	Sector 5
A_{t-1}	1	1	1	1	1
$w_{j,t-1}$	5	5	5	5	5
$A_{j,t-1}$	1	1	1	1	1
$p_{j,t-1}$	8	8	8	8	8
$p^m_{j,t-1}$	8	8	8	8	8
$z_{i,j,t-1}$	0.0125	0.0125	0.0125	0.0125	0.0125
$A_{i,j,t-1}$	1	1	1	1	1
$a_{i,j,t-1}$	1	1	1	1	1
$K_{i,j,t-1}$	1	1	1	1	1

Table 3: Initial conditions (set equally among economies, sectors and firms)

B Mathematical Appendix

This appendix aims to detail some of the intermediate manipulations used for the presentation of the model. We will then explicit here the computation of the balance of payment constraint, the path allowing us to deduce the expressions for the GDP growth rate as the expression for GDP.

B.1 The computation of sectors exports and imports growth rates

Starting from the expression for sector j's exports :

$$X_{j,c,t} = s_{j,w,t}(Y_{w,t})^{\alpha_c} z_{j,c,t}$$

we obtain the following expression for the growth rate of exports in this sector:

$$\frac{\Delta X_{j,c,t}}{X_{j,c,t-1}} = \frac{\Delta s_{j,w,t}}{s_{j,w,t-1}} + \alpha_c \frac{\Delta Y_{w,t}}{Y_{w,t-1}} + \frac{\Delta z_{j,c,t}}{z_{j,c,t-1}}$$

Given the expression for $s_{j,w,t}$ as defined in the model we obtain:

$$\frac{\Delta s_{j,w,t}}{s_{j,w,t-1}} = (\epsilon_j - 1) \frac{\Delta Y_{w,t}}{Y_{w,t-1}}$$

Given the expression for $z_{j,c,t}$ we obtain that:

$$
\begin{aligned}
&= \sum_i z_{i,j,c,t} \\
&= \sum_i z_{i,j,c,t-1}\left[1+\phi\left(\frac{E_{i,j,c,t}}{\bar{E}_{j,t}}-1\right)\right] \\
&= \sum_i z_{i,j,c,t-1} + \sum_i \phi\left[z_{i,j,c,t-1}\left(\frac{E_{i,j,c,t}}{\bar{E}_{j,t}}-1\right)\right]
\end{aligned}
$$

$$
\begin{aligned}
z_{j,c,t} &= z_{j,c,t-1} + \phi\left(\frac{\sum_i z_{i,j,c,t-1}E_{i,j,c,t}}{\bar{E}_{j,t}} - z_{j,c,t-1}\right) \\
&= z_{j,c,t-1} + z_{j,c,t-1}\phi\left(\frac{\sum_i \frac{z_{i,j,c,t-1}}{z_{j,c,t-1}}E_{i,j,c,t}}{\bar{E}_{j,t}}-1\right) \\
&= z_{j,c,t-1} + z_{j,c,t-1}\phi\left(\frac{E_{j,c,t}}{\bar{E}_{j,t}}-1\right) \text{ with } E_{j,c,t} = \sum_i \frac{z_{i,j,c,t-1}}{z_{j,c,t-1}}E_{i,j,c,t}
\end{aligned}
$$

Hence :

$$
\frac{\Delta z_{j,c,t}}{z_{j,c,t-1}} = \phi\left(\frac{E_{j,c,t}}{\bar{E}_{j,t}}-1\right)
$$

Thus we obtain the following expression for sector j's export growth rate:

$$
\frac{\Delta X_{j,c,t}}{X_{j,c,t-1}} = (\alpha_c + \epsilon_j - 1)\frac{\Delta Y_{w,t}}{Y_{w,t-1}} + \phi\left(\frac{E_{j,c,t}}{\bar{E}_{j,t}}-1\right)
$$

Symmetrically, we can compute import's growth rate. Starting from the expression for sector j's imports :

$$
M_{j,c,t} = s_{j,c,t}(Y_{c,t})^{\beta_c}(1-z_{j,c,t})
$$

we obtain the following expression for the growth rate of exports in this sector:

$$
\frac{\Delta M_{j,c,t}}{M_{j,c,t-1}} = \frac{\Delta s_{j,c,t}}{s_{j,c,t-1}} + \beta_c\frac{\Delta Y_{c,t}}{Y_{c,t-1}} + \frac{\Delta(1-z_{j,c,t})}{(1-z_{j,c,t-1})}
$$

Given the expression for $s_{j,c,t}$ as defined in the model we obtain:

$$
\frac{\Delta s_{j,c,t}}{s_{j,w,t-1}} = (\epsilon_j - 1)\frac{\Delta Y_{c,t}}{Y_{c,t-1}}
$$

Given the expression for $z_{j,c,t}$ we obtain that:

$$
\begin{aligned}
&= \frac{1-z_{j,c,t}-(1-z_{j,c,t-1})}{1-z_{j,c,t-1}} \\
&= \frac{z_{j,c,t-1}-z_{j,c,t}}{1-z_{j,c,t-1}} \\
\frac{\Delta(1-z_{j,c,t})}{1-z_{j,c,t-1}} &= \frac{z_{j,c,t-1}-\left[z_{j,c,t-1}+z_{j,c,t-1}\phi\left(\frac{E_{j,c,t}}{\bar{E}_{j,t}}-1\right)\right]}{1-z_{j,c,t-1}} \\
&= -\frac{z_{j,c,t-1}}{1-z_{j,c,t-1}}\phi\left(\frac{E_{j,c,t}}{\bar{E}_{j,t}}-1\right) \\
&= -b_{j,c,t-1}\phi\left(\frac{E_{j,c,t}}{\bar{E}_{j,t}}-1\right) \text{ with } b_{j,c,t-1} = \frac{z_{j,c,t-1}}{1-z_{j,c,t-1}}
\end{aligned}
$$

Thus we obtain the following expression for sector j's import growth rate:

$$
\frac{\Delta M_{j,c,t}}{M_{j,c,t-1}} = (\beta_c + \epsilon_j - 1)\frac{\Delta Y_{c,t}}{Y_{c,t-1}} - b_{j,c,t-1}\phi\left(\frac{E_{j,c,t}}{\bar{E}_{j,t}}-1\right)
$$

B.2 The computation of the balance of payment constraint

Let us first consider $X_{c,t}$ as the total exports of the domestic economy. It can be expressed as the sum of sectoral exports:

$$X_{c,t} = \sum_j p_{j,c,t} X_{j,c,t}$$

It's growth rate can then be expressed as follows:

$$\frac{\Delta X_{c,t}}{X_{c,t-1}} = \frac{\Delta \left(\sum_j p_{j,c,t} X_{j,c,t} \right)}{X_{c,t-1}} = \frac{\sum_j \Delta \left(p_{j,c,t} X_{j,c,t} \right)}{\sum_j p_{j,c,t-1} X_{j,c,t-1}} = \frac{\sum_j X_{j,c,t-1} \Delta p_{j,c,t} + p_{j,c,t-1} \Delta X_{j,c,t}}{\sum_j p_{j,c,t-1} X_{j,c,t-1}}$$

Through some minor manipulation $\frac{\Delta X_{c,t}}{X_{c,t-1}}$ can be expressed as follows:

$$\frac{\Delta X_{c,t}}{X_{c,t-1}} = \sum_j \frac{\Delta p_{j,c,t}}{p_{j,c,t-1}} \frac{p_{j,c,t-1} X_{j,c,t-1}}{\sum_j p_{j,c,t-1} X_{j,c,t-1}} + \sum_j \frac{\Delta X_{j,c,t}}{X_{j,c,t-1}} \frac{p_{j,c,t-1} X_{j,c,t-1}}{\sum_j p_{j,c,t-1} X_{j,c,t-1}}$$

Given the expression of $X_{j,c,t}$ as defined in the model:

$$X_{j,c,t} = s_{j,w,t} (Y_{w,t})^{\alpha} z_{j,c,t}$$

we can then simplify the expression for $\frac{p_{j,c,t-1} X_{j,c,t-1}}{\sum_j p_{j,c,t-1} X_{j,c,t-1}}$ as follows:

$$\begin{aligned}
\frac{p_{j,c,t-1} X_{j,c,t-1}}{\sum_j p_{j,c,t-1} X_{j,c,t-1}} &= \frac{p_{j,c,t-1} s_{j,w,t-1} (Y_{w,t-1})^{\alpha} z_{j,c,t-1}}{\sum_j p_{j,c,t-1} s_{j,w,t-1} (Y_{w,t-1})^{\alpha} z_{j,c,t-1}} \\
&= \frac{(Y_{w,t-1})^{\alpha} p_{j,c,t-1} s_{j,w,t-1} z_{j,c,t-1}}{(Y_{w,t-1})^{\alpha} \sum_j p_{j,c,t-1} s_{j,w,t-1} z_{j,c,t-1}} \\
&= \frac{p_{j,c,t-1} s_{j,w,t-1} z_{j,c,t-1}}{\sum_j p_{j,c,t-1} s_{j,w,t-1} z_{j,c,t-1}} = e_{j,c,t-1}
\end{aligned}$$

The same procedure can be applied for imports. Hence starting from

$$M_{c,t} = \sum_j p^m_{j,c,t} M_{j,c,t}$$

One can easily show that:

$$\frac{\Delta M_{c,t}}{M_{c,t-1}} = \sum_j \frac{p^m_{j,c,t-1} M_{j,c,t-1}}{\sum_j p^m_{j,c,t-1} M_{j,c,t-1}} \frac{\Delta M_{j,c,t}}{M_{j,c,t-1}} + \sum_j \frac{p^m_{j,c,t-1} M_{j,c,t-1}}{\sum_j p^m_{j,c,t-1} M_{j,c,t-1}} \frac{\Delta p^m_{j,c,t}}{p^m_{j,c,t-1}}$$

Given the expression of $M_{jc,,t}$ as defined in the model:

$$M_{j,c,t} = s_{j,c,t} (Y_{c,t})^{\beta} (1 - z_{j,c,t})$$

we can then deduce that:

$$\frac{p^m_{j,c,t-1} M_{j,c,t-1}}{\sum_j p^m_{j,c,t-1} M_{j,c,t-1}} = \frac{p^m_{j,c,t-1} s_{j,c,t-1} (1 - z_{j,c,t-1})}{\sum_j p^m_{j,c,t-1} s_{j,c,t-1} (1 - z_{j,c,t-1})} = i_{j,c,t-1}$$

Hence, the expression for the balance of payment constraint written as:

$$\frac{\Delta X_{c,t}}{X_{c,t-1}} = \frac{\Delta M_{c,t}}{M_{c,t-1}}$$

can be expressed as follows:

$$\sum_j i_{j,c,t-1} \frac{\Delta M_{j,c,t}}{M_{j,c,t-1}} + \sum_j i_{j,c,t-1} \frac{\Delta p^m_{j,c,t}}{p^m_{j,c,t-1}} = \sum_j e_{j,c,t-1} \frac{\Delta X_{j,c,t}}{X_{j,c,t-1}} + \sum_j e_{j,c,t-1} \frac{\Delta p_{j,c,t}}{p_{j,c,t-1}}$$

B.3 From balance of payment constraint to the expression of GDP growth rate

Starting from balance of payment constraint:

$$\sum_j i_{j,c,t-1}\frac{\Delta M_{j,c,t}}{M_{j,c,t-1}}+\sum_j i_{j,c,t-1}\frac{\Delta p^m_{j,c,t}}{p^m_{j,c,t-1}}=\sum_j e_{j,c,t-1}\frac{\Delta X_{j,c,t}}{X_{j,c,t-1}}+\sum_j e_{j,c,t-1}\frac{\Delta p_{j,c,t}}{p_{j,c,t-1}}$$

We can substitute for each sectors, imports and exports growth rates by the expressions defined above in the paper. We then obtain the following expression for the balance of payment constraint:

$$\sum_j i_{j,c,t-1}\left[(\beta_c+\epsilon_j-1)\frac{\Delta Y_{c,t}}{Y_{c,t-1}}-b_{j,c,t-1}\phi\left(\frac{E_{j,c,t}}{\bar{E}_{j,t}}-1\right)\right]+\sum_j i_{j,c,t-1}\frac{\Delta p^m_{j,c,t}}{p^m_{j,c,t-1}}$$

$$=\sum_j e_{j,c,t-1}\left[(\alpha_c+\epsilon_j-1)\frac{\Delta Y_{w,t}}{Y_{w,t-1}}+\phi\left(\frac{E_{j,c,t}}{\bar{E}_{j,t}}-1\right)\right]+\sum_j e_{j,c,t-1}\frac{\Delta p_{j,c,t}}{p_{j,c,t-1}}$$

Given that the sum among j of $i_{j,c,t-1}$ and $e_{j,c,t-1}$ equal one, hence the previous expression can be re-written as follows:

$$\left(\beta_c+\sum_j i_{j,c,t-1}\epsilon_j-1\right)\frac{\Delta Y_{c,t}}{Y_{c,t-1}}-\phi\sum_j i_{j,c,t-1}b_{j,c,t-1}\left(\frac{E_{j,c,t}}{\bar{E}_{j,t}}-1\right)+\sum_j i_{j,c,t-1}\frac{\Delta p^m_{j,c,t}}{p^m_{j,c,t-1}}$$

$$=\left(\alpha_c+\sum_j e_{j,c,t-1}\epsilon_j-1\right)\frac{\Delta Y_{w,t}}{Y_{w,t-1}}+\phi\sum_j e_{j,c,t-1}\left(\frac{E_{j,c,t}}{\bar{E}_{j,t}}-1\right)+\sum_j e_{j,c,t-1}\frac{\Delta p_{j,c,t}}{p_{j,c,t-1}}$$

Starting from this expression of the balance of payment constraint, we can then isolate an expression for the GDP growth rate:

$$\frac{\Delta Y_{c,t}}{Y_{c,t-1}}=\gamma_{c,t-1}\frac{\Delta Y_{w,t}}{Y_{w,t-1}}+\lambda_{c,t-1}\phi\left[\sum_j \theta_{j,c,t-1}\left(\frac{E_{j,c,t}}{\bar{E}_{j,t}}-1\right)\right]+\lambda_{c,t-1}\sum_j \kappa_{j,c,t-1}$$

With

$$\gamma_{c,t-1}=\frac{\alpha_c+\sum_j e_{j,c,t-1}\varepsilon_j-1}{\beta+\sum_j i_{j,c,t-1}\varepsilon_j-1}$$

$$\lambda_{c,t-1}=\frac{1}{\beta+\sum_j i_{j,c,t-1}\varepsilon_j-1}$$

$$\theta_{j,c,t-1}=e_{j,c,t-1}+i_{j,c,t-1}b_{j,c,t-1}$$

$$\kappa_{j,c,t-1}=e_{j,c,t-1}\frac{\Delta p_{j,c,t}}{p_{j,c,t-1}}-i_{j,c,t-1}\frac{\Delta p^m_{j,c,t}}{p^m_{j,c,t-1}}$$

C Figures

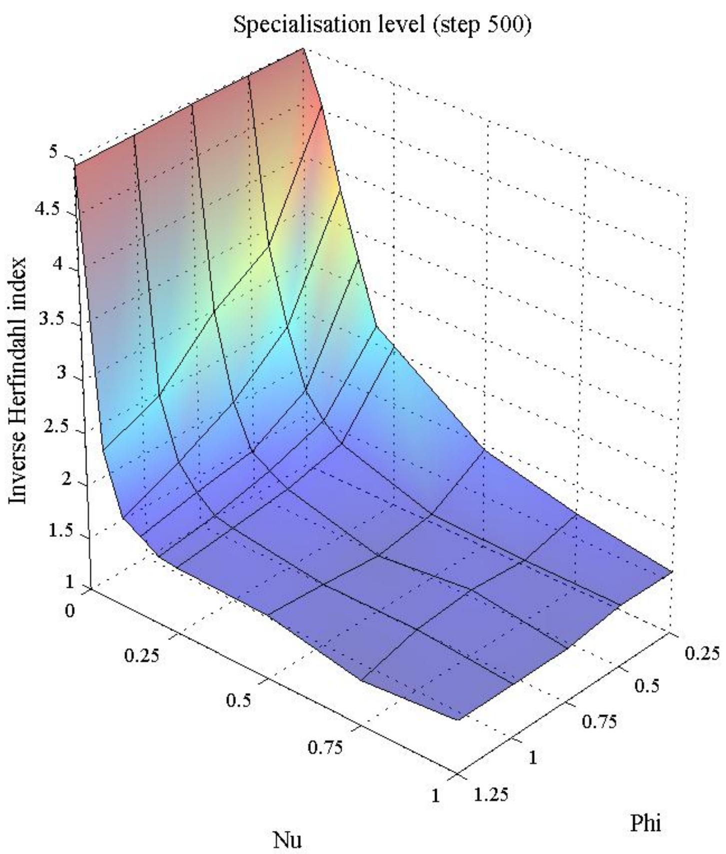

Figure 1: Specialisation levels and selection parameters (ϕ vs. ν)

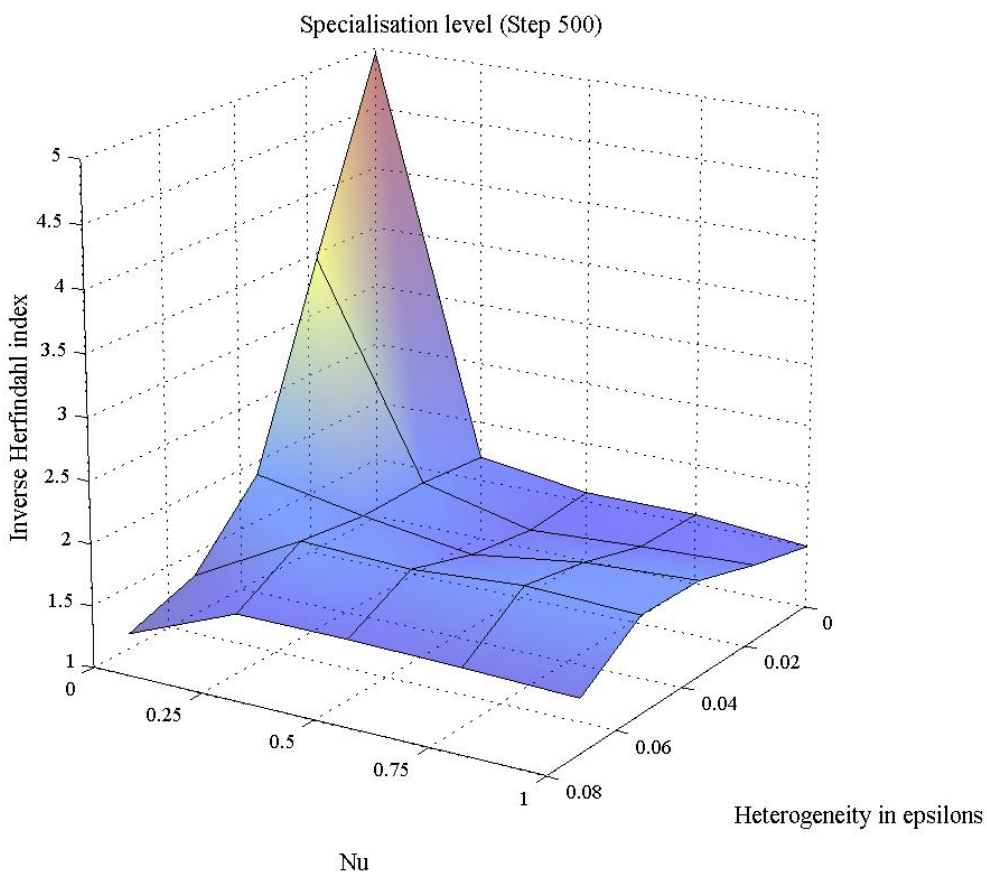

Figure 2: Specialisation levels with heterogenous income elasticities ($StdDev(\varepsilon_j)$ vs. ν)

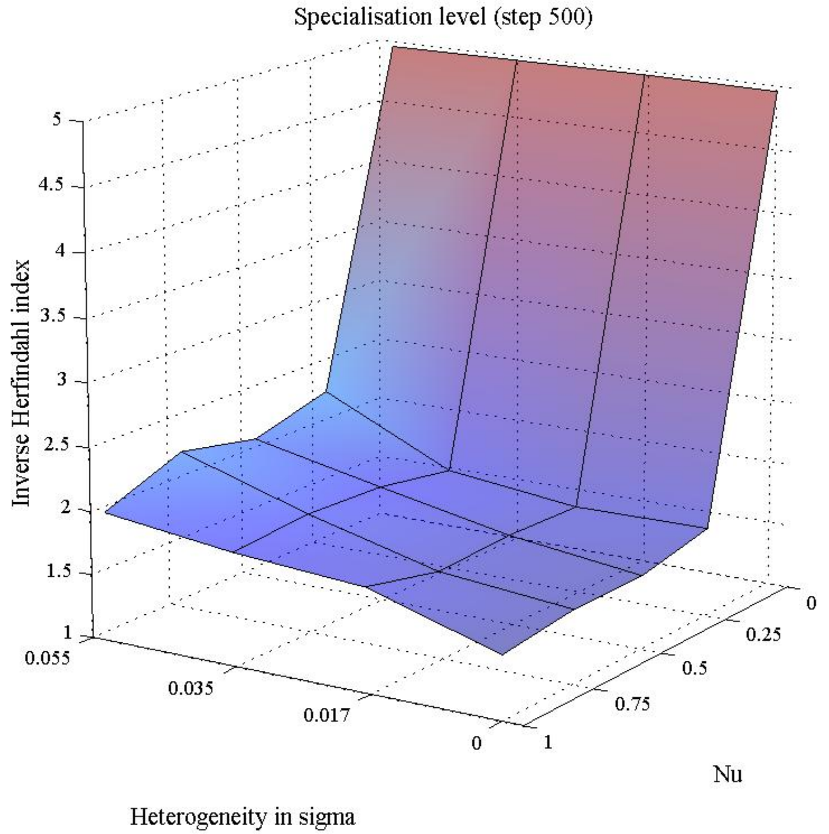

Figure 3: Specialisation levels with heterogenous technological opportunities ($StdDev(\sigma_j)$ vs. ν)

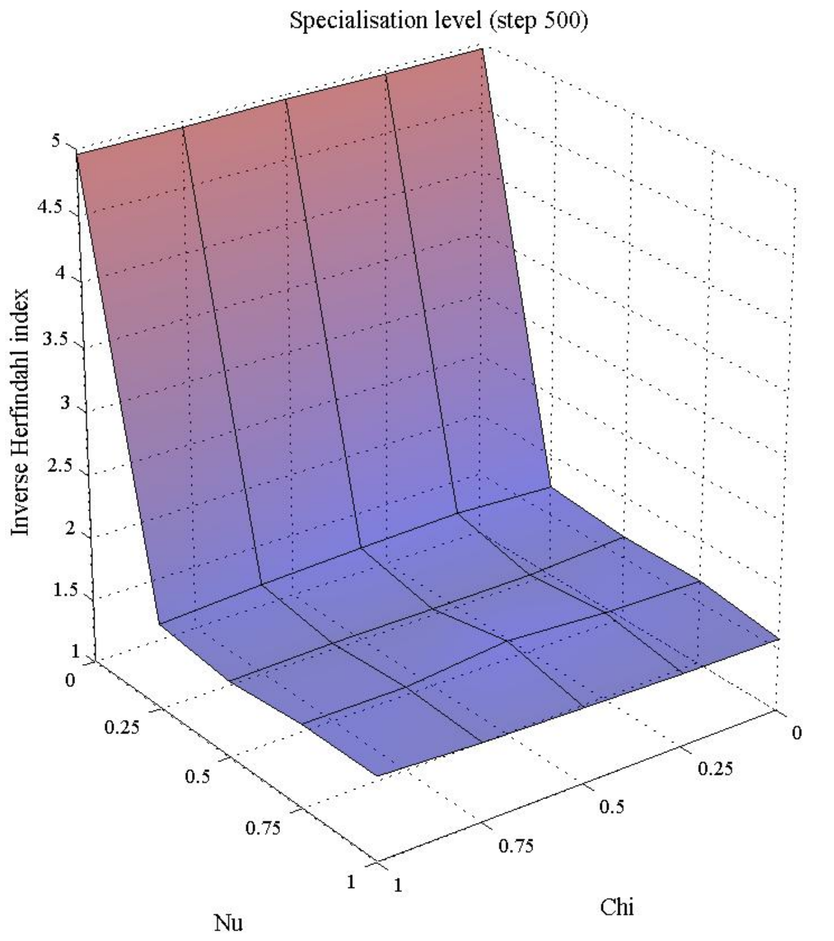

Figure 4: Specialisation levels and the appropriability of spillovers (χ)

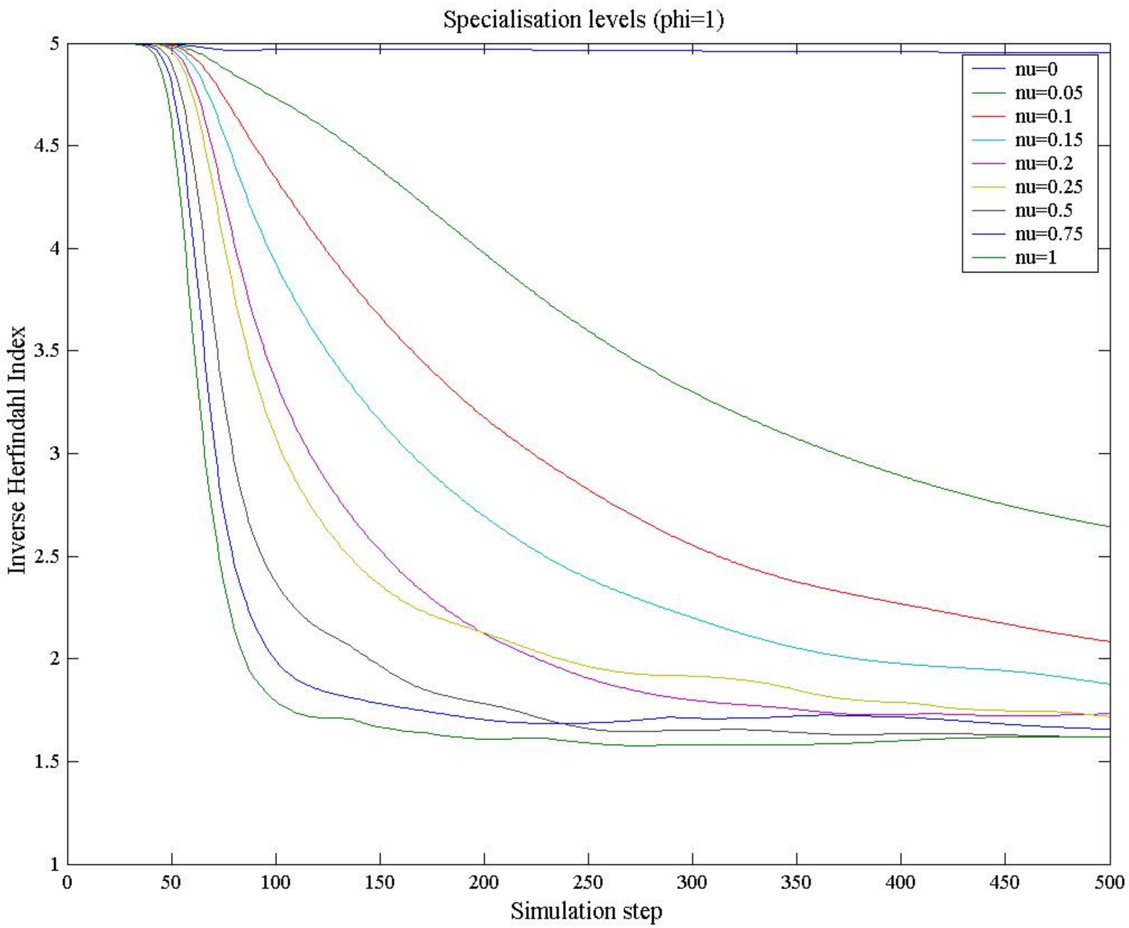

Figure 5: Specialisation dynamics with various values of ν

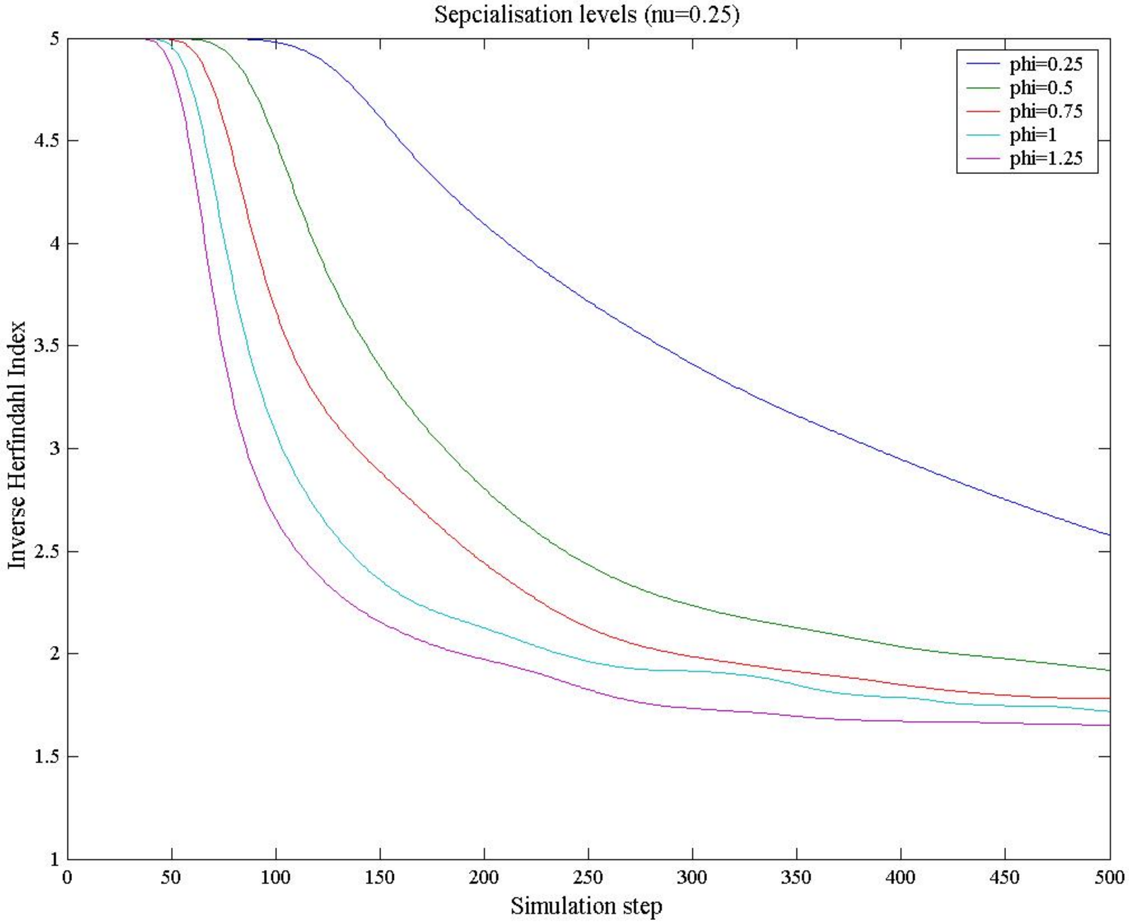

Figure 6: Specialisation dynamics and price elasticity (ϕ)

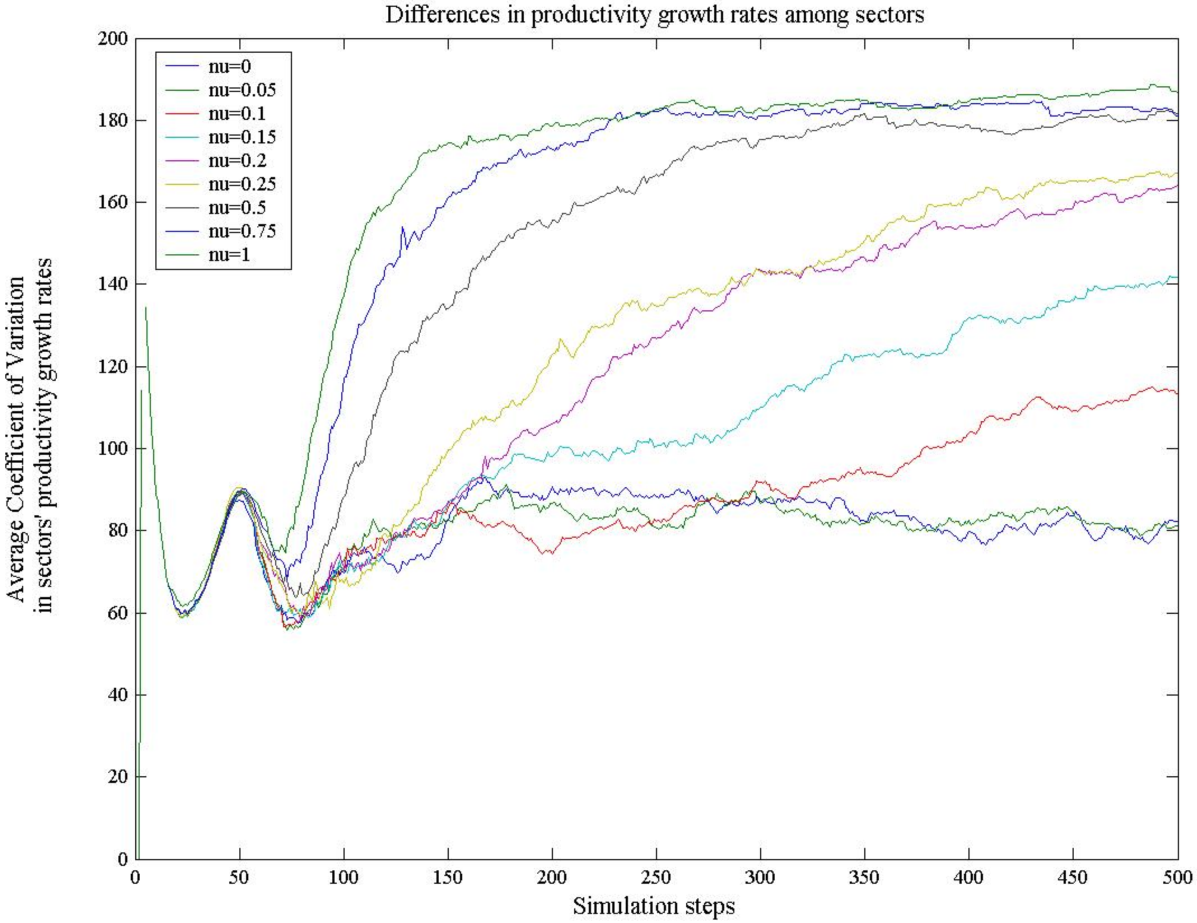

Figure 7: Inter-sector differences in productivity growth rates and inter-sector selection through wages

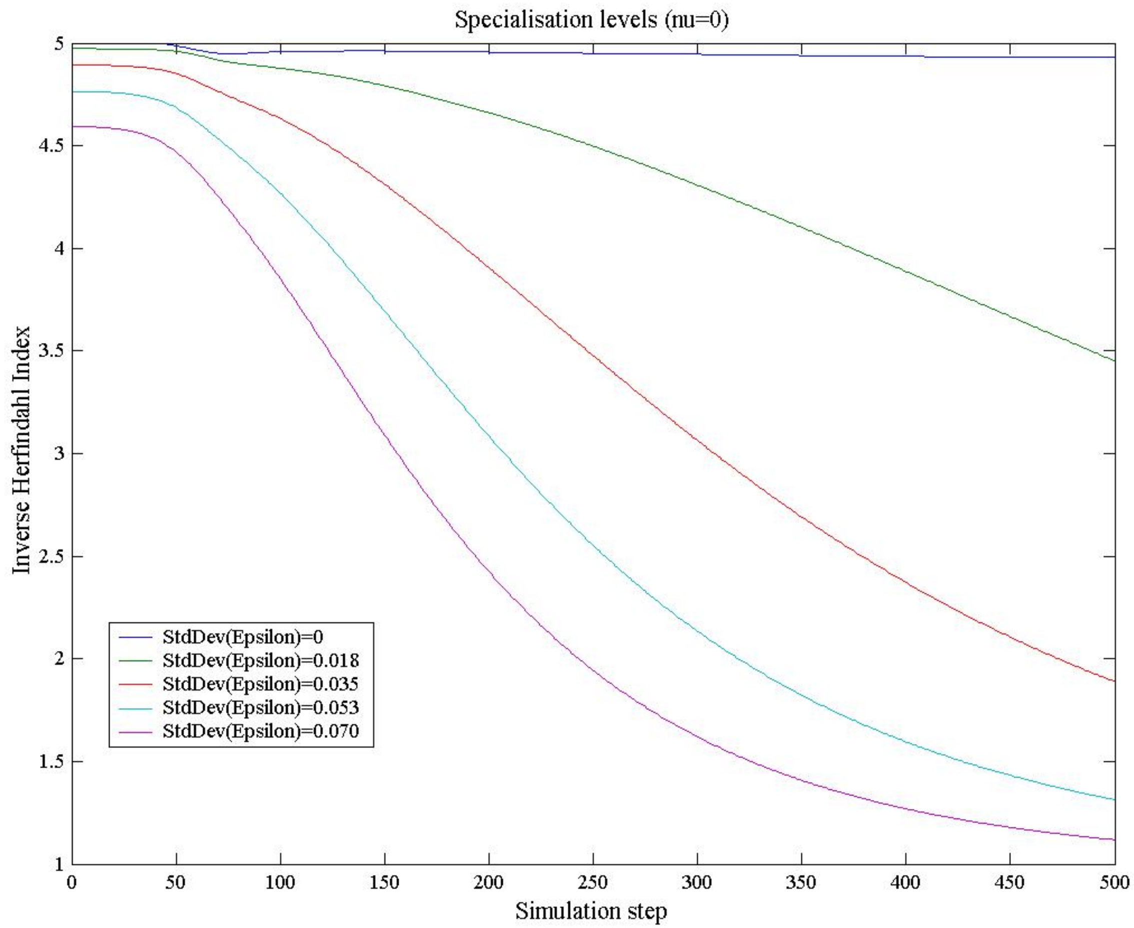

Figure 8: Specialisation dynamics and heterogeneity in income elasticity

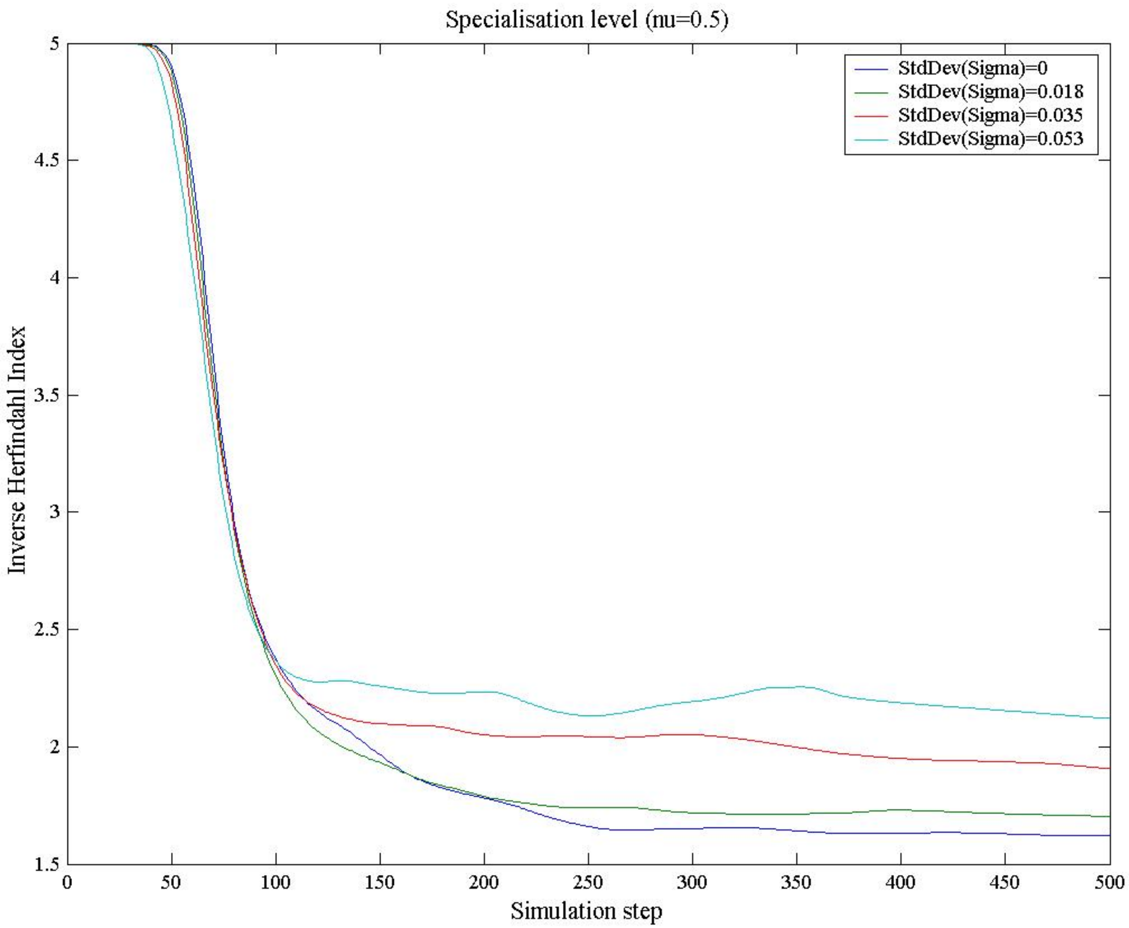

Figure 9: Specialisation dynamics and heterogeneity in technological opportunities

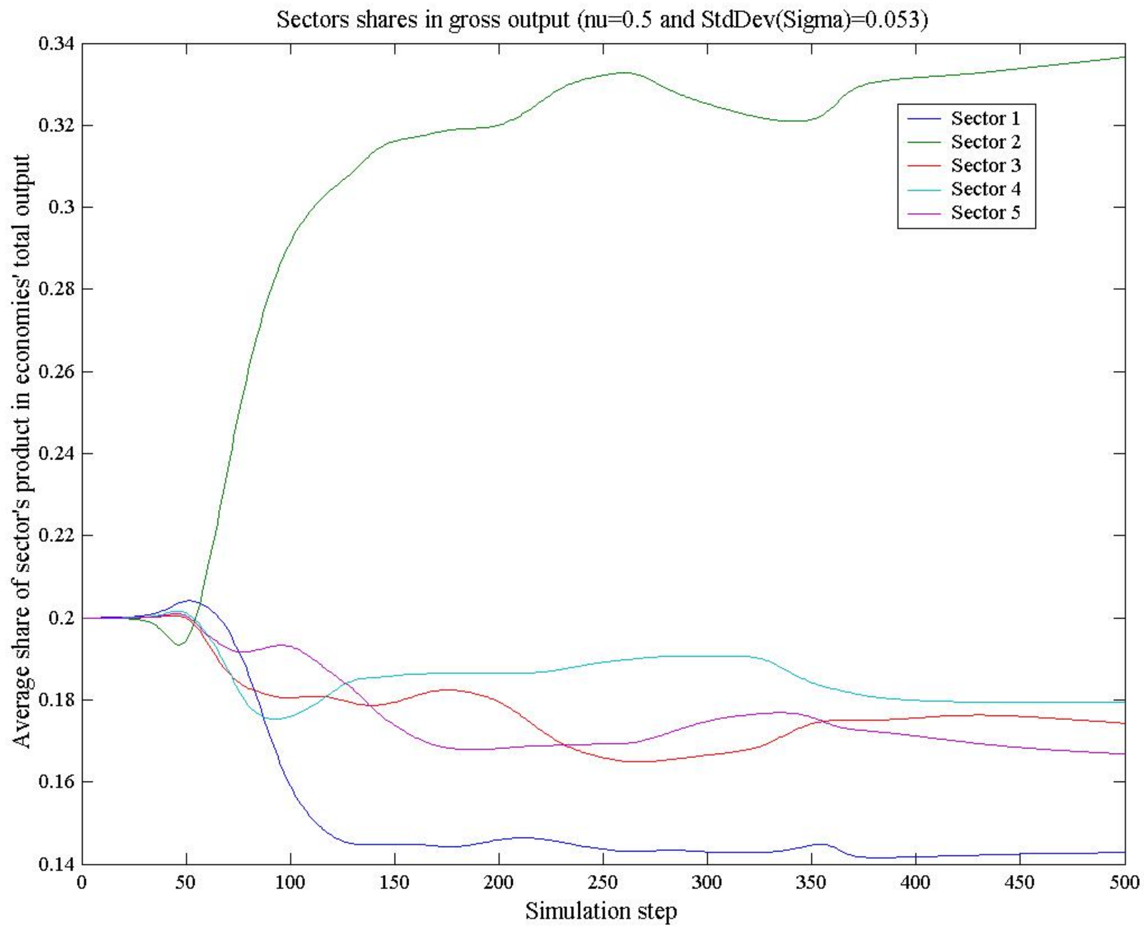

Figure 10: Sectors' share in total production (Average over the 5 economies)

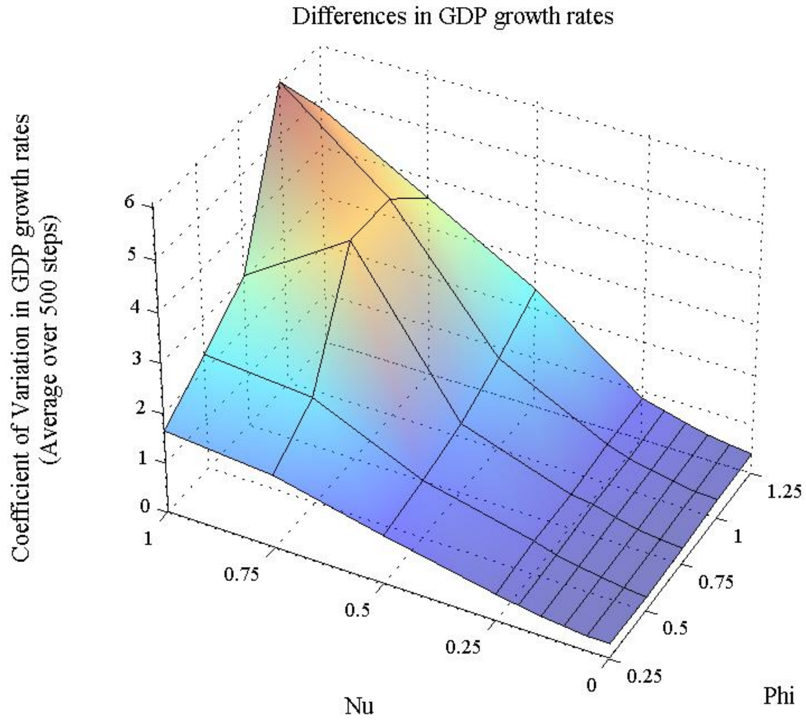

Figure 11: Differences in GDP growth rates (Average over 500 steps) and selection parameters (ϕ vs. ν)

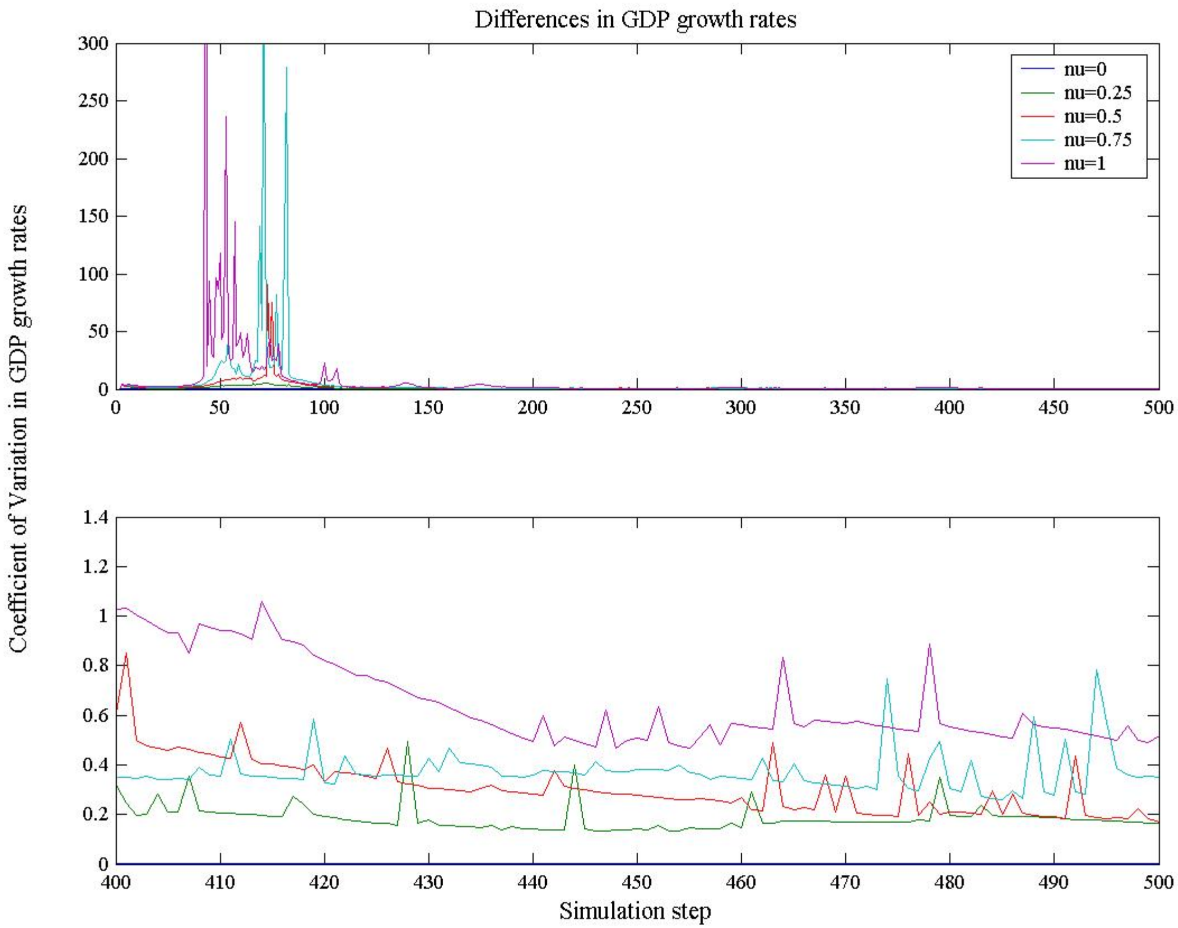

Figure 12: Differences in GDP growth rates with various values of ν

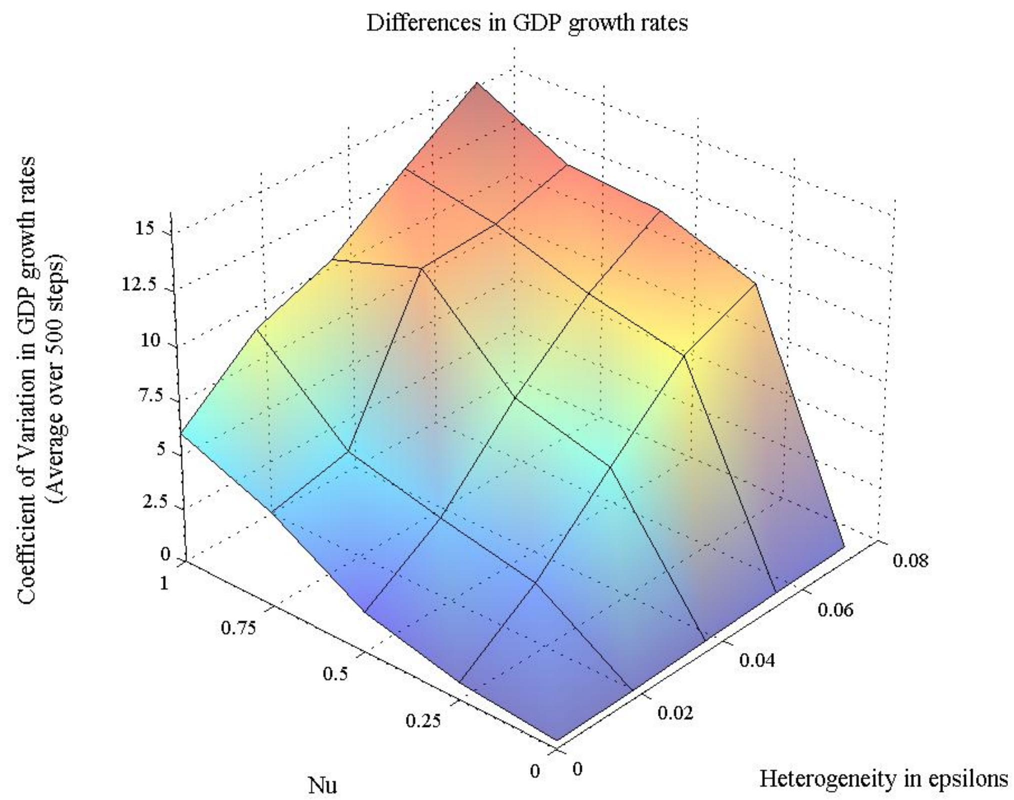

Figure 13: Differences in GDP growth rates (Average over 500 steps) with heterogenous income elasticities ($StdDev(\varepsilon_j)$ vs. ν)

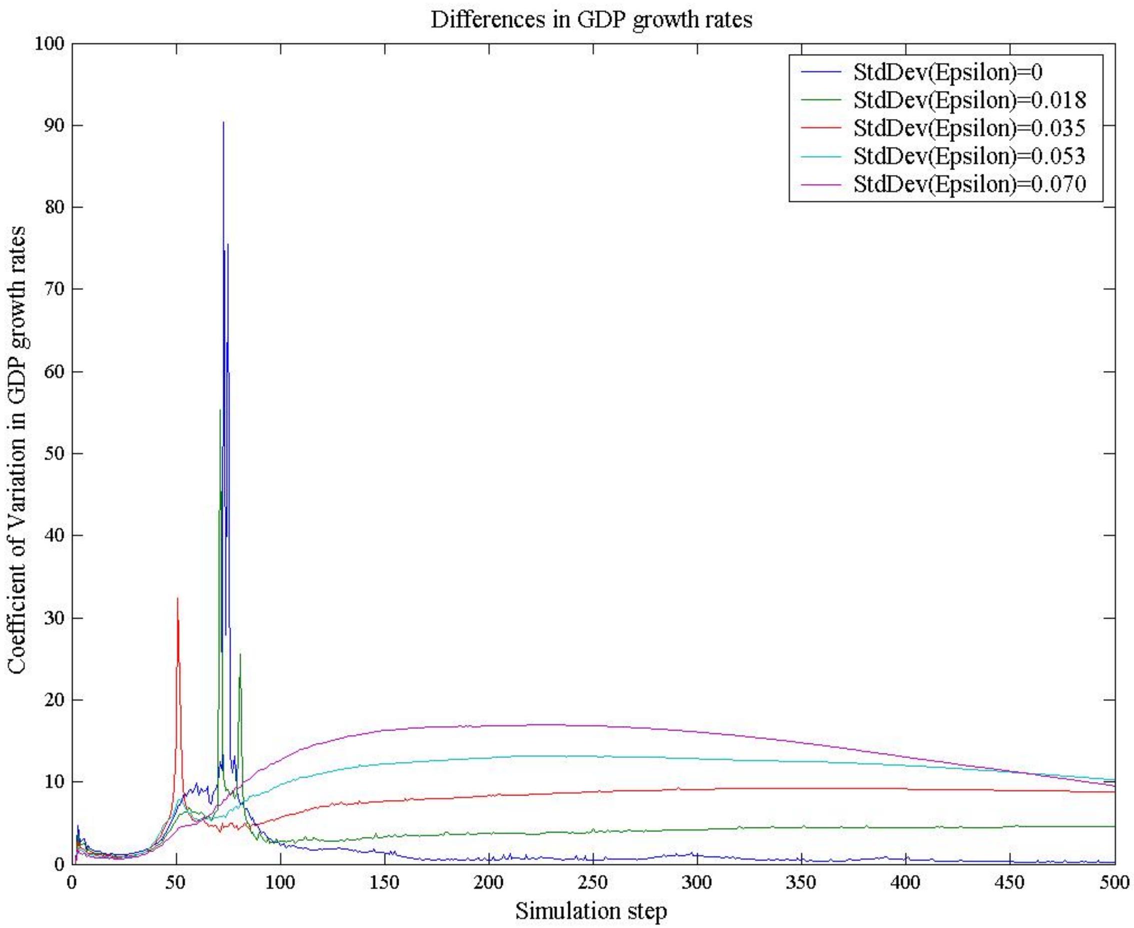

Figure 14: Differences in GDP growth rates with heterogenous income elasticities ($StdDev(\varepsilon_j)$)

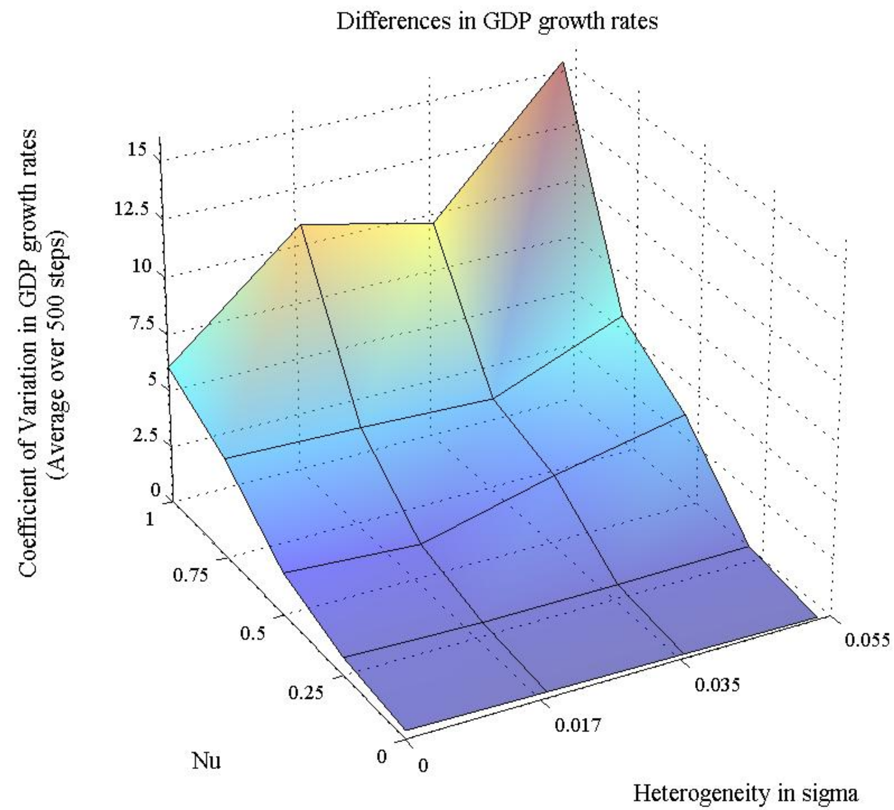

Figure 15: Differences in GDP growth rates (Average over 500 steps) with heterogenous technological opportunities ($StdDev(\sigma_j)$ vs. ν)

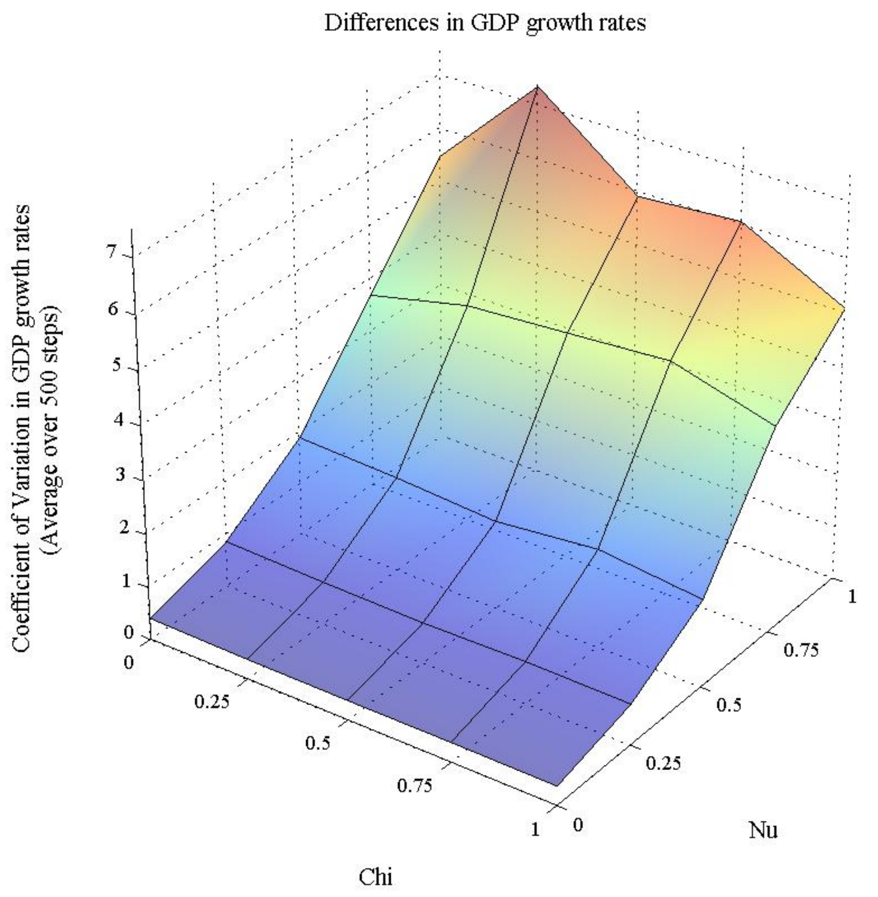

Figure 16: Differences in GDP growth rates (Average over 500 steps) and the appropriability of technological spillovers