Yeast vs. Mushrooms:  
A Note on Harberger’s  
“A View of the Growth Process”

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2004/03  
February 2004
Yeast vs. Mushrooms: A Note on Harberger’s “A Vision of the Growth Process”*

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January 30, 2004

What is the most realistic description of the growth process? Does growth spread evenly across industries, or is it an unbalanced process? Diverse research traditions give support to the former view. There is substantial historical evidence about technological complementarities and the role of pervasive technologies as engines of growth.1 Some of the mechanisms behind such evidence have been formalized in General Purpose Technology models.2 New Growth Theory provides further mechanisms in support of this view, such as knowledge externalities, human capital accumulation, and scale effects.3 Last, but not least, the Schumpeterian concept of bandwagon effect following fundamental innovations also provides an example of a mechanism leading to even patterns of growth across sectors.4

On the other hand, the discovery of a huge residual, within the growth accounting approach, soon fostered a vast empirical research on the sources of

*Comments and support by Alfonso Gambardella are gratefully acknowledged. The usual disclaimers apply.

1Such as machine tools (Rosenberg, 1963), the steam engine (Rosenberg and Trajtenberg, 2001), electricity (Devine, 1983), and ICTs (David, 1990). See also Rosenberg (1976), and David and Wright (1999).

2See Bresnahan and Trajtenberg (1995), and the works collected in Helpman (1998).


4See Schumpeter (1934). Romer (1990) and Aghion and Howitt (1992) have built growth models with Schumpeterian features.
economic growth.\textsuperscript{5} Within this stream of literature, Harberger’s Presidential Address at the 1998 Annual Meeting of the American Economic Association (Harberger, 1998) makes clear that the growth process is probably not as even as it is often described: the effect of common shocks looks negligible as compared to industry- and firm-specific causes of productivity change.

This conclusion is based on the analysis of US firm- and industry-level TFP growth rates, 5-year averages from 1948 to 1991, yielding the following set of stylized facts:

(i) A small-to-modest fraction of industries can account for 100\% of aggregate Real Cost Reduction (RCR), defined as the product between Total Factor Productivity (TFP) growth and Initial Value Added. In other words, sectoral contributions to aggregate TFP growth are concentrated in few industries;

(ii) The complementary fraction of industries contains both “winners” (i.e. those with positive productivity growth) and “losers” (with negative productivity growth). Their TFP contributions sum to zero;

(iii) Losers are a very important part of the picture, and contribute greatly to the observed dynamics of aggregate TFP. Indeed, when the aggregate TFP contribution to RCR is relatively small, the cumulative total of the positive contributions is a large multiple of that aggregate. Conversely, when the aggregate contribution is large, that multiple tends to be smaller;

(iv) There is little evidence of persistence from period to period in the leadership of TFP performances.

As a way of putting his evidence into perspective, Harberger proposes the following dichotomy between visions of the growth process:

a \textit{yeast} vision which “fits best with very broad and general externalities, like externalities linked to the growth of the total stock of knowledge or human capital or brought about by economies of scale tied to the scale of the economy as a whole”; and

a \textit{mushroom} vision which “fits best with a vision of real cost reductions stemming from 1001 different causes”.\textsuperscript{6}

Harberger argues that the set of stylized facts presented in his paper is supportive of the mushroom vision, thereby casting doubts on the empirical

\textsuperscript{5}Abramowitz (1956), Solow (1957), Denison (1967), and Jorgenson and Griliches (1967) are among the pioneers in this tradition.

\textsuperscript{6}Harberger (1998), pp. 4 and 5.
relevance of broad externalities as engines of growth. This because, if growth
was mainly driven by broad externalities, then contributions to aggregate
RCR ought to be quite evenly distributed across sectors. However, as shown
by facts (i) and (ii) above, they are pretty concentrated. Concentration is
indeed very clear from Harberger’s Lorenz-like curves, the so-called sunrise
(or sunset) diagrams.\footnote{Harberger (1998), p. 8, Fig. 2}

In this note, we argue that Harberger’s evidence is not incompatible with
a yeast vision of the growth process. More specifically, we show that, if
one allows for heterogeneity in elasticities of sectoral TFP to shocks from
other sectors, Harberger’s evidence can be reconciled with the yeast view.
Our result is that, if such elasticities are heterogeneous across sectors, then
concentration in the sectoral contributions to aggregate RCR can occur, even
if sectoral TFP growth processes are completely driven by a shock stemming
from a single sector.

Formally, suppose the economy is composed of \( n > 1 \) industries, and
that total output of the generic industry \( i \) at time \( t \), \( Q_{it} \), can be represented
through the following Cobb-Douglas function:

\[
Q_{it} = A_{it} K_{it}^{\alpha} L_{it}^{1-\alpha}
\] (1)

where \( K_i \) and \( L_i \) are capital and labour inputs, and \( A_{it} \) is TFP. As-
sume that TFP in sector \( i \) is a function of industry specific shocks, \( Z_k, k = 1, \ldots, i, \ldots, n, \) affecting all industries in the economy:

\[
A_{it} = A_i(Z_{1t}, Z_{2t}, \ldots, Z_{it}, \ldots, Z_{nt})
\] (2)

where \( A_i \) is continuous and twice differentiable with respect to all its
arguments. Given (2), the growth rate of TFP in sector \( i \) reads:

\[
g_i = \frac{1}{A_i} \frac{dA_i}{dt} = \frac{1}{A_i} \sum_{j=1}^{n} \frac{\partial A_i}{\partial Z_j} \frac{dZ_{jt}}{dt}
\] (3)

Multiplying and dividing each term in the summation by \( Z_{jt} \), we obtain:

\[
g_i = \sum_{j=1}^{n} \varepsilon_j a_j
\] (4)
where $\varepsilon_i^j \equiv \left. \frac{\partial A_i}{\partial Z_{jt}} \right|_{t}$ is the elasticity of TFP in industry $i$ with respect to the shock $Z_{jt}$, and $a_j \equiv \frac{dZ_{jt}}{dt} Z_{jt}$, assumed constant over time for simplicity. Notice that the elasticity term $\varepsilon_i^j$ measures the response of the growth rate of TFP in sector $i$ to a change in $Z_j$. When $\varepsilon_i^j$ is low (high), sector $i$ has a low (high) reaction with respect to the shock $Z_j$ affecting sector $j$. Accordingly, its TFP growth rate reacts poorly (greatly) to such a shock.

Note that the formulation given in (1)-(4) is very general and includes, as special cases:

(a) the “mushrooms” process, in which TFP growth in industry $i$ depends only on an idiosyncratic shock $Z_{it}$:

$$g_i = \varepsilon_i^i a_i$$  \hspace{1cm} (5)

(b) a “pure yeast” process, such that TFP in each industry is entirely due to a shock stemming from a given sector $z$:

$$g_i = \varepsilon_i^z a_z$$  \hspace{1cm} (6)

(c) a hybrid case, in which sectoral TFP growth depends both on common and idiosyncratic factors:

$$g_i = \varepsilon_i^i a_i + \varepsilon_i^z a_z$$  \hspace{1cm} (7)

Within the above framework, we show that concentration in aggregate TFP growth contributions can occur even in a pure yeast economy. Graphically, this means that the Lorenz curve can differ from the 45 degrees line. To show this, we first find a necessary and sufficient condition for inequality in the general case of (4), then we study such a condition in the pure yeast case, shown in (6).

Harberger’s sunrise diagrams display the cumulative sum of sectoral Initial Value Added shares on the horizontal axis and, on the vertical axis, the cumulative sum of contributions of individual industries to aggregate RCR (Initial Value Added multiplied by TFP growth). Formally, for industry $i$ we have, on the horizontal axis,

$$q_i = \frac{\sum_{j=1}^{i} Q_j}{\sum_{j=1}^{i} Q_j}$$  \hspace{1cm} (8)
and, on the vertical axis,

\[ x_i = \frac{\sum_{j=1}^{i} Q_j g_j}{\sum_{j=1}^{n} Q_j g_j} \]  \hspace{1cm} (9)

Multiplying and dividing the last expression by \( \sum_{j=1}^{i} Q_j \sum_{j=1}^{n} Q_j \) we obtain:

\[ x_i = \frac{G_i}{G} q_i \]  \hspace{1cm} (10)

where \( G_i \equiv \frac{\sum_{j=1}^{i} Q_j g_j}{\sum_{j=1}^{n} Q_j} \) and \( G \equiv \frac{\sum_{j=1}^{n} Q_j g_j}{\sum_{j=1}^{n} Q_j} \) are, respectively, the weighted average of TFP growth rates over the first \( i \) sectors and the aggregate TFP growth rate. Notice that in (10) the cumulative sum of sectoral contributions to RCR is a linear function of the cumulative shares of Initial Value Added, i.e. the variable on the horizontal axis. The ratio \( G_i / G \) is thus the “local” slope of the Lorenz-like curve. Indeed, concentration is zero if and only if \( |G_i / G| = 1 \forall i \).\(^8\) Hence, the vector \( G = [G_1, G_2, ..., G_i, ..., G] / G \) is a measure of concentration. Equidistribution therefore requires:

\[ \frac{\sum_{j=1}^{i} Q_j g_j}{\sum_{j=1}^{n} Q_j} = \frac{\sum_{j=1}^{n} Q_j g_j}{\sum_{j=1}^{n} Q_j}, \forall i \]  \hspace{1cm} (11)

For \( i = 1 \), (11) becomes:

\[ g_1 = G \]  \hspace{1cm} (12)

For \( i = 2 \), we must have:

\[ \frac{Q_1}{Q_1 + Q_2} G + \frac{Q_2}{Q_1 + Q_2} g_2 = G, \text{ i.e., } g_2 = G \]  \hspace{1cm} (13)

Via induction, we conclude that a necessary condition for zero concentration is:

\(^8\)Actually, in sunset diagrams the ratio \( G_i / G \) equals the cumulative rate of TFP growth. Without loss of generality, we normalize this to one.
\( g_i = g_j \ \forall i, j \quad (14) \)

Condition (14) is also sufficient. To see why, suppose \( g_i = g_j = g, \ \forall i, j \), and plug into expressions for \( G_i \) and \( G \). This yields \( G_i = G = g, \ \forall i \). Hence, we conclude that

\[
\frac{G_i}{G} = 1 \iff g_i = g_j, \ \forall i, j \quad (15)
\]

The above implies that concentration in the sectoral contributions to aggregate RCR arises whenever industries are heterogeneous in terms of TFP growth rates.\(^9\) Since \( g_i = \sum_{j=1}^{n} \varepsilon_i^j a_j \), the necessary and sufficient condition for concentration in contributions to aggregate RCR is:

\[
\sum_{j=1}^{n} \varepsilon_i^j a_j \neq \sum_{j=1}^{n} \varepsilon_k^j a_j \quad (16)
\]

for at least one couple \((i, k)\in \{1, \ldots, n\}, i \neq k\).

Let us now restrict our analysis to the pure yeast case in (6). In such a case, condition (16) boils down to:

\[
\varepsilon_i^z \neq \varepsilon_k^z \quad (17)
\]

for at least one couple \((i, k)\in \{1, \ldots, n\}, i \neq k\).

Hence, concentration in the pure yeast case arises if and only if at least two sectors have different elasticities of TFP with respect to the shock stemming from sector \(z\).

Condition (17) is suggesting that the evidence on concentration is not enough in discriminating between the yeast and the mushrooms visions of the growth process. Growth might look like mushrooms even if a common component drives productivity growth of all industries. Indeed, concentration might be due to heterogeneity in elasticities of sectoral TFP to shocks from other sectors. Different mechanisms may be behind heterogeneity, such

\(^9\)The statement follows from inequality being the logical complement of equality. Indeed, the negation of (14) implies that the opposite is true and therefore that \( |G_i - G| \neq 1 \) for at least one \(i\). Henceforth, we shall consider the opposite of condition (14) as the necessary and sufficient condition for concentration in contributions to real cost reduction.
as cross-sectoral differences in size, in human capital stocks, and in absorptive capacities.\textsuperscript{10}

The latter seems to us a very appealing candidate in this respect. This because absorptive capacity is not merely an individual property: it is rather a feature of the interaction between firms, in the same as well as in different industries. Who absorbs what, from whom, and how, all matter here. Hence, in our search for the best description of the growth process, we might need to go beyond the simple dichotomy discussed in this note.

References


\textsuperscript{10}For a thorough analysis of the concept of absorptive capacity, see Cohen and Levinthal (1989, 1990).


