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Abstract

This paper investigates the relevance of financial and economic variables as determinants of firm defaults. Our analysis is not limited to publicly traded companies but extends to a large sample of limited liability firms. We consider size, growth, profitability and productivity together with a standard set of financial indicators. Non parametric tests allow to asses to what extent defaulting firms differ from the non-defaulting group. Bootstrap probit regressions confirm that economic variables play both a long and short term effect. Our findings are robust with respect to the inclusion of Distance to Default and risk ratings among the regressors.

JEL codes: C14, C25, D20, G30, L11.

Keywords: firm default, financial indicators, selection and growth dynamics, kernel densities, stochastic equality, bootstrap probit regressions, Distance to Default.

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1 Introduction

This paper presents an empirical analysis of firm default exploiting information on distress events occurring in a large panel of Italian firms. The fundamental motivation is to assess whether the inclusion of economic variables alongside traditional financial ones allows to better investigate the causes of firms' default, possibly improving the chance to correctly distinguish "healthy" firms from those at risk of distress.

Inside the broadly defined field of financial economics, and in particular in the analysis of firm default conducted in the context of credit ratings construction, firms' distress is typically conceived to be primarily determined by poor financial conditions, especially in the short run before default occurs. Purely industrial factors tend to receive less attention and sometimes turn completely left out of the analysis, possibly presuming that their effect is more relevant several periods before default and in any case already embedded into shorter run financial performances.[1] At the same time, it is well understood that the probability to stay in the market as well as the financial stability of a firm is deeply intertwined with the ability to perform well along the economic dimensions of its operation. At least as long as market frictions or other institutional factors are affecting the extent and the speed to which economic performances get perfectly reflected into financial structure and financial conditions, it is possible that looking exclusively at financial indicators cannot offer but a partial account of the main determinants of default. Starting from similar considerations Grunert et al. (2005) propose an "augmented" version of a standard financial model of default prediction which also includes two "soft" non-financial characteristics (managerial quality and market position) among the regressors. The aim of their exercise, asking whether non-financial indicators can improve upon default predictions based on banks' internal rating systems, is however different and to some extent narrower than what we are pursuing here. Yet, an empirical verification of whether economic variables contain any further explanatory power over financial indicators is largely lacking.

We believe that there is wide room for further improvements and we foresee our understanding of default events can benefit from the inclusion of a sensible subset of economic indicators in the analysis. Admittedly, the decision of which variables could play the most prominent role is not a straightforward one. However, modern theories of firm-industry dynamics offer a solid guidance to our attempt. Indeed, despite different schools of thought exist on the theory of the firm, a shared view is that survival and growth are eventually determined by the action of a selection mechanism which occurs through interaction and competition in the market, and operates on the economic characteristics of heterogeneous firms. Typically, one has models where a certain level of productivity/efficiency or, more generally, certain bundles of capabilities/competences represent the necessary conditions to remain in the market. Then, the interplay between firm characteristics and the environment, shaped by the competition forces driving the market, results into differential profitability levels and, ultimately, into exit or growth events. Whatever the specific theory one might discuss, there is strong agreement that productivity, profitability and size-growth dynamics represent the key levels of corporate performance along the selection process.[2] These variables therefore form the set of economic/industrial characteristics which we add to standard financial indicators in our

[1] This is a tradition of analysis followed since the classical studies by Beaver (1966) and Altman (1968). See Altman and Saunders (1998) and Crouhy et al. (2000) for a more complete review of the literature.

[2] The same message is consistent with broad sense neoclassical models of firm-industry dynamics (see, for instance, Jovanovic, 1982; Ericson and Pakes, 1995; Melitz, 2003), as well with models originating from the evolutionary tradition (see Winter, 1971; Nelson and Winter, 1982).

attempt to investigate the determinants of default events.[3]

A remarkable feature of the present analysis rests in its wide scope. While typical studies in firm default focus on large publicly traded companies, our studies cover almost 20,000 limited liabilities firms active in manufacturing, very different in size and type of activity. These makes our study highly representative of the dynamics of Italian industry. Moreover, the present analysis can also be seen as a contribution to the industrial economic literature concerned with the determinants of firm exit. Indeed, in a large part of empirical studies in industrial dynamics "exit" can only be approximated via firms' disappearing from the sample due to a reduction in number of employees or annual turnover below certain threshold which determine if a firm is included in the databases typically employed. In our case "exit" is identified by a truly economic event, default, which by definition signal an at least temporary stop of firms' operation.

We offer two specific contributions. First, we explore the heterogeneities possibly existing both within and across defaulting *vis á vis* non-defaulting firms along each dimension considered. We estimate the empirical distribution of their financial and economic characteristics, and employ non-parametric tests for stochastic equality of the two groups, also looking at variation of results near to default *vs.* further away from it. Second, and in accordance with the main purpose of the paper, we estimate a series of probit models of default probability, allowing us to identify which are the main determinants of default once the effects of economic and financial factors are allowed to simultaneously interplay. Bootstrap techniques allow for robust estimates of the relevant coefficients, and a set of model evaluation criteria is also introduced, enabling to discern if default prediction accuracy is improved when economic characteristics are added to financial factors. The findings remain valid when we add, among the regressors, a variant of Distance to Default (Merton, 1974) and an official credit rating index. This testify in favor of the strong robustness of our conclusions with respect to the inclusion of dimensions which we might not directly capture through available financial and economic variables.

Our findings confirm the conjecture that explicitly adding economic indicators can enhance the understanding of the process leading to firms' default. The analysis of empirical distributions reveals that defaulting and non-defaulting firms display important differences, not only along financial characteristics, but along economic variables too. Further, results from bootstrap probit regressions reveal that economic characteristics exert a significant impact on the probability of default, complementary and additional with respect to the contribution of financial indicators. Notably, such an effect remains significant even near to default, when one would instead assume that economic factors have been already embodied into financial conditions. These results do not depend from sectoral specificity at the level of 2-Digit industries, do not vary when we include Distance to Default predictor among the regressors, and remain unchanged after extending the models to include credit ratings.

The work is organized as follows. Section 2 presents a detailed description of the dataset. A first, descriptive comparison of defaulting *vs* non-defaulting firms is provided in Section 3, based on kernel estimates of the empirical distribution of economic and financial variables in the two groups of firms. Section 4 further explores the issue by means of more formal statistical tests of distributional equality. In Section 5 we then tackle bootstrap probit estimates of default probability, focusing on whether the addition of economic variables can improve

[3]A huge empirical literature has highlighted the positive effect exerted on survival by the technological characteristics of the firms (see Agarwal and Audretsch, 2001, for a review), like R&D expenditures or patents. However we lack the necessary data to include these further dimensions in our analysis (see details in Section 2).

explanatory and predictive power of the models, as compared to a benchmark specification where only financial indicators are used. Robustness of results with respect to inclusion of Distance to Default and credit ratings is then tested in Section 6. Finally, Section 7 concludes suggesting some interpretations.

2 Data, variables and sample selection

We employ the database maintained by the Centrale dei Bilanci (CeBi), which contains financial statements and balance sheets of virtually all Italian *limited liability* firms. Italian Civil Law enforces the public availability of the annual accounting for this category of firms. CeBi collects and organizes this information, performing initial reliability checks. Included firms operates in all industrial sectors and no threshold is imposed on their size. This represents a remarkable advantage over other firm level panels, which typically cover only firms reporting more than a certain number of employees. The dataset as such is quite rich and detailed, and appears particularly suitable for the analysis of both large and small-medium sized firms.

For the sake of the present work we have access to manufacturing data over the period 1998-2003. For each firm, the following variables are available to us: Total Sales (S), Value Added (VA, i.e. sales minus costs of inputs), Number of Employees (L), Cost of Labour, Gross Operating Income (GOI, as value added minus cost of labour), Gross Tangible Assets (K), Total Assets, Return on Investment (ROI), Return on Equity (ROE), Leverage (as the ratio of assets over the sum of shareholders' equity plus annual income after taxes), Interest Expenses (IE), and Financial-Debt-to-Sales ratio (FD/S).

From this list, we select our indicators of firms' financial and economic characteristics, as follows. Concerning the financial side, although we can build less indicators than one typically finds in studies of bankruptcy prediction, we can anyway capture the "strands of intuition" (see Carey and Hrycay, 2001) lying behind the type of indicators usually employed. Interest Expenses (IE) provide a flow measure of the annual costs bore by firms to repay debt; Leverage is a standard indicator of the relative balance between external *vs* internal financing; finally, the Financial-Debt-to-Sales ratio (FD/S) gives a stock measure of overall exposure, scaled by size of the firm. On the other hand, concerning economic characteristics, we are able to include in the analysis measures of size, growth, profitability and productivity, that is, the four basic levels which theoretical models as well as empirical research in industrial economics suggest to capture the crucial measures of firm performances. Several proxies can be in principle adopted to measure each of these dimensions, each proxy capturing complementary aspects of the same phenomenon. First, concerning firm size, revenue based (sales or value added) measures appear to be more suited to have a relationship with default than alternative "physical" (in terms of employment or capital) measures. Thus, we measure size in terms of Total Sales (S), and, accordingly, the growth rate of Total Sales (g^S) is used to measure firm growth. Second, concerning profitability, we want a proxy of the margins generated by the industrial or operational activities of the firms, which is the level we are interested into, avoiding definitions of income or profits influenced by financial strategies and taxation. Accordingly, we define profitability in terms of the return on sales (ROS), i.e. the ratio between Gross Operating Income and Total Sales, which also avoids complications pertaining estimation of value and costs of capital, requiring more reliable data than we have. Finally, productive efficiency is captured by a standard index of labor productivity, measured in terms of value added per employee. Various reasons suggest to prefer this measure over alternative multi-factor proxies of efficiency. The main motivation is that, due to mentioned problems in measuring capital and its costs, we want to keep a correspondence between productivity and profitability measures. Consider also that estimates of multi-factor productivities are model dependent and impose strong assumptions (more on this point in Dosi and Grazzi, 2006; Bottazzi et al., 2008). Overall, although multi-factor measures obviously present in principle the advantage to capture efficiency associated with other inputs, and capital in particular, it is not clear whether, in practice, the advantages of using multi-factor proxies overcomes the drawbacks.

Though limited, the list of selected variables is sufficiently rich to give a relevant account of both the economic and the financial side of firm operation. In the robustness checks of Section 6 these variables are supplemented with a measure of Distance to Default, which we compute from annual reports' variables along the lines discussed in Bharath and Shumway (2008), and by a credit rating index produced by CeBi itself (see Section 6 for details on these measures).

Accounting data from the CeBi database are then matched with a dummy variable taking on value 1 when a firm incurs default at the end of the period (in either 2003 or 2004), and 0 otherwise. These default events are provided by an Italian bank only for those firms which were among its customers during the sample period. This implies that default status can be identified with certainty only for a subset of the available CeBi sample. It is therefore likely that our dataset understates distressed firms with respect to default frequency rates actually occurring in the population. Consequently, incorrect estimates might arise in regression analysis due to a classical "choice-based sample bias", which is a common problem in studies of distress prediction.[4] A first basic strategy that we apply in order to overcome this potential drawback is to restrict the analysis only to those firms reporting at least 1 Million Euro of Total Sales in each year. This threshold is indeed a reasonable lower bound for the customer' size of our reference bank, and, therefore, excluding firms below this level of sales enhances comparability between the default sub-sample and the rest of the dataset. On the top of this, and relatedly, a second cleaning step is to remove all those firms reporting only one employee. This cut allows to focus the study only on firms displaying at least a minimal level of structure and operation, and it is also intended to exclude self-employment, which is represented by single-employee businesses in the CeBi data.

Our final sample includes 19628 manufacturing firms. The first two columns of Table 1 show the number of firms and default events by 2-Digit sectors, according to the NACE (Rev. 1.1) industrial classification.[5] The last two columns, instead, compare default rates in our data with default rates in the reference population of Italian limited liability firms (average between 2003 and 2004), as officially reported by the association of the Italian Chambers of Commerce. As shown, under-weighting of distressed sample is only partially solved by the implemented cleaning. Since this problem is likely to be particularly harmful for probit estimates, the analyses in Section 5 and Section 6 also applies a bootstrap sampling procedure designed to make default frequencies equivalent to the actual default rates observed at the population level.

[4]Zmijevski (1984) analyzes this point in depth. Notice however that default events tend to be over-represented in the samples typically employed in that literature, an opposite situation as compared to the problem we must face here.

[5]*Nomenclature génerale des Activités économiques dans les Communautés Europénnes*, NACE, is the standard at European level, and perfectly matches, at the 2-Digit level, with the International Standard Industrial Classification, ISIC.

Sectors	Number of firms	Number of defaults	Default rate in the Sample	Default rate in the population
15 - Food products & beverages	2008	9	0.0045	0.0302
17 - Manufacture of textiles	1544	14	0.0091	0.0474
18 - Wearing apparel; dressing; dyeing of fur	673	9	0.0134	0.0511
19 - Leather; luggage, & footwear	762	15	0.0197	0.0375
20 - Manufacture of wood	396	2	0.0051	0.0249
21 - Pulp & paper products	540	2	0.0037	0.0298
22 - Publishing, printing & recorded media	749	6	0.0080	0.0377
24 - Chemical products	1122	2	0.0018	0.0383
25 - Rubber and plastic products	1176	6	0.0051	0.0338
26 - Other non-metallic mineral products	1142	5	0.0044	0.0309
27 - Manufacture of basic metals	672	2	0.0030	0.0378
28 - Metal products (except machinery & equip.)	2473	14	0.0057	0.0280
29 - Machinery and equipment n.e.c.	2916	23	0.0079	0.0352
31 - Electrical machinery and apparatus n.e.c.	835	7	0.0084	0.0361
32 - Radio, TV & communication equip.	280	6	0.0214	0.0473
33 - Medical, precision & optical instruments	472	1	0.0021	0.0399
34 - Motor vehicles, trailers & semi-trailers	442	8	0.0181	0.0364
35 - Other transport equipment	234	6	0.0256	0.0382
36 - Furniture; manufacturing n.e.c.	1184	10	0.0084	0.0341
Total	19628	147	0.0075	

Table 1: The first three columns report respectively the number of firms, the number of defaults default rates in the sample, computed at 2-Digit sectoral level. The last column displays the corresponding default rates in the population of Italian limited liability firms (averages between 2003 and 2004 – Source: the association of the Italian Chambers of Commerce, UNIONCAMERE).

3 Descriptive analysis

This section analyzes if and to what extent the economic and financial characteristics of defaulting firms differ from the rest of the sample. We compare the empirical distribution of the relevant variables across the two groups of firms. To take account of the possible intertemporal variation, we present results at different time distance to default, comparing the estimates in the first available year, 1998, with the estimates obtained in the last year before default occurs, 2002. We apply non-parametric techniques, which do not impose any *a priori* structure to the data, thereby allowing to take a fresh look at the heterogeneities possibly existing both within and across the two groups of firms. The descriptive nature of this analysis is supplemented by formal statistical tests for distributional equality, performed in the next Section.

3.1 Economic characteristics

We start with the comparison of firm size. In the two panels of Figure 1 we plot, on a double logarithmic scale, the kernel density of firm size (S) estimated for defaulting and non-defaulting firms in 1998 and in 2002. The actual values of S for each defaulting firm are depicted in the bottom part of each plot.[6]

[6]Here, as well as in the following, estimates are performed applying an Epanenchnikov kernel, and the bandwidth is set following the "optimal rules" suggested in Silverman (1986), Section 3.4.

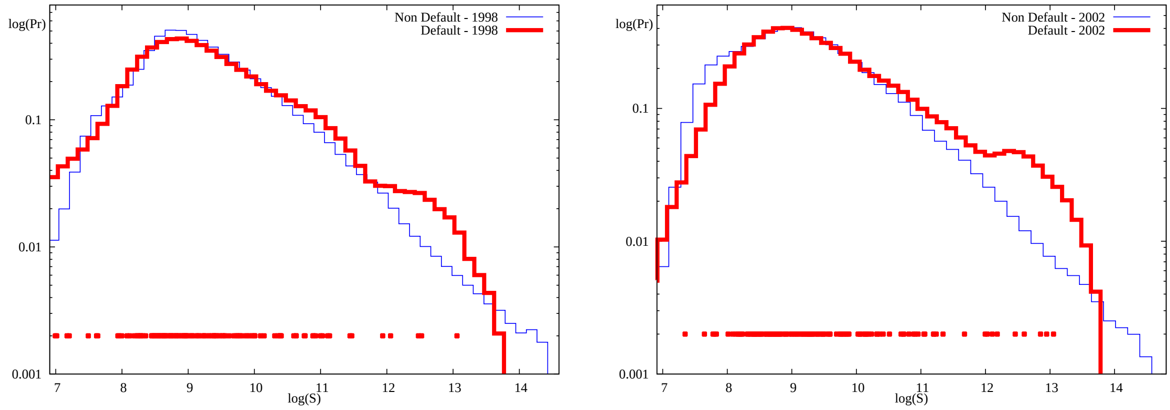

Figure 1: Empirical density of Total Sales (S) in 1998 (**left**) and 2002 (**right**): Defaulting *vs* Non-Defaulting firms.

First look at 1998. Somewhat contrary to what one might conjecture, defaulting firms are neither less heterogeneous nor smaller with respect to the rest of the sample. The two densities are indeed very similar: supports are comparable, and the shapes are both right-skewed, an empirical result repeatedly found in the literature on firm size distribution. Actually, the right part of defaulting firms density suggests that default events are more frequent among medium-big sized firms rather than at small sizes. Estimates for 2002, in the right panel, show that these properties remain stable over time, as the default is approaching. The upper tail of the defaulting firms' distribution is indeed even heavier as compared to 1998. A formal non-parametric test of multi-modality (Silverman, 1986) cannot reject the presence of bimodality in the distributions of defaulting firms (with a p-score of 0.72 for 1998 and of 0.63 for 2002). Such differences in the tail behavior are anyhow due to relatively few very big firms (see the dots at the bottom of the plots). Instead, in the central part of the densities, where most of the observations are placed, the overlap is almost perfect. Overall, the evidence is therefore suggestive that there is no clearcut relationship between size and the event of default: operating above a certain size threshold does not seem to provide any relevant warranty in preventing default.

Next we focus on firms' growth. Figure 2 shows kernel densities of Total Sales growth rates, g^S, for defaulting and non-defaulting firms. Given the initial year of the sample is 1998, the first available data point is for 1999. This is shown in the left panel, while 2002 is depicted on the right graph. As before, actual values of g^S for defaulting firms are reported below the estimated densities.

Defaulting firms do not appear to significantly differ from the rest of the sample when considering the portions of supports where most of the probability mass is concentrated (approximately between -0.5 and 0.5). In this interval, the distributions are crossing each other, and the estimated shapes are very similar, independently from the different time distance to default considered. One difference emerges regarding the variability of growth episodes: in 2002, that is closer to the default event, the width of the supports spanned by the defaulting group is sensibly narrower, especially in the right part of the support. This gets also mirrored in the tails (outside the interval $[-0.5, 0.5]$), where we however observe some differences across the two groups. In both years considered, left tail behavior is similar across the two groups, suggesting similar occurrence of extremely bad growth records. Conversely, only few default-

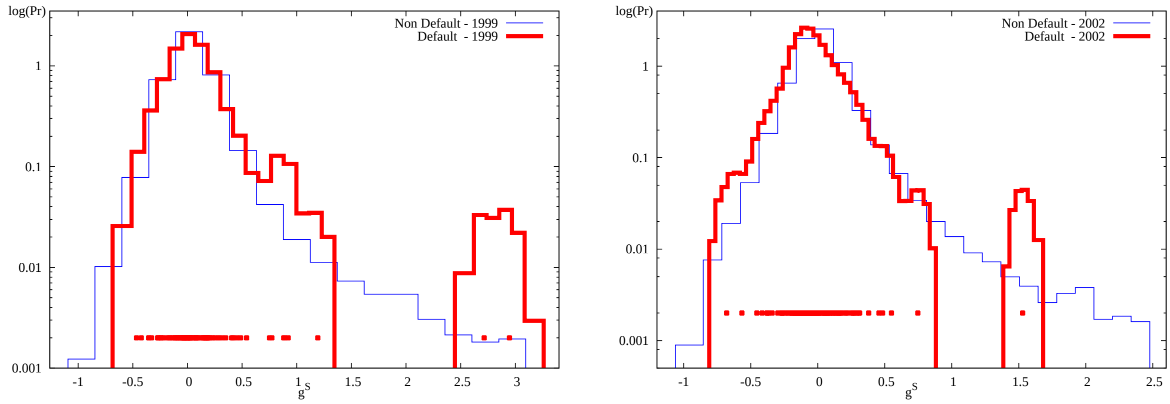

Figure 2: Empirical density of Total Sales Growth (g^S) in 1999 (**left**) and 2002 (**right**): Defaulting *vs* Non-Defaulting firms.

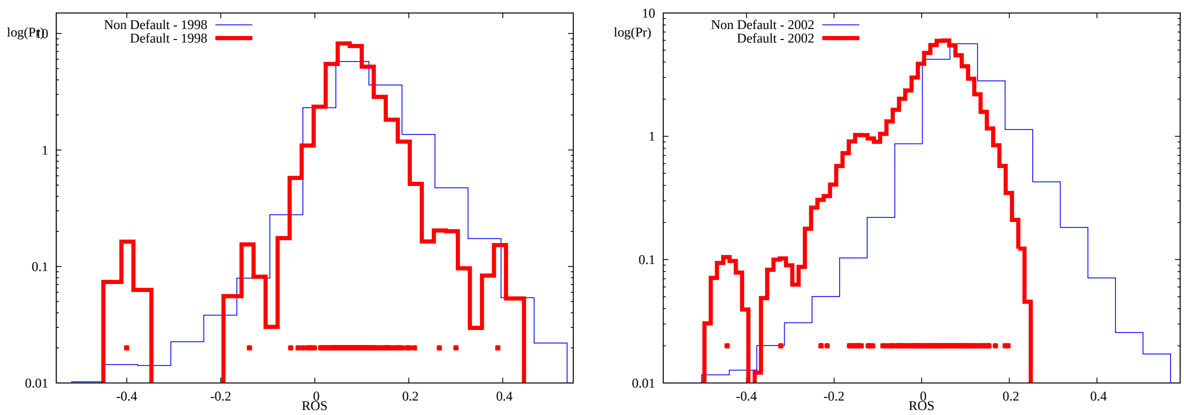

Figure 3: Empirical density of Profitability (ROS) in 1998 (**left**) and 2002 (**right**): Defaulting *vs* Non Defaulting Firms.

ing firms are responsible for the peaks present at the top extremes. The different sizes of the two compared samples is likely to play a role in this respect. Similarly to what noted for size, however, these tail patterns concern a very low number of firms, and therefore they offer too weak evidence to conclude that one of the two groups is significantly outperforming the other.

We then repeat the same exercise with profitability performance. Figure 3 reports kernel estimates of ROS densities in 1998 and 2002. The two groups of firms tend this time to differ, as defaulting firms perform clearly worse than the rest of the sample, especially for positive values of ROS. In 1998, the two distributions are substantially overlapping in the negative half of the support, while the density of defaulting firms lies constantly below that of the other group in the positive half. The same ranking gets reinforced in 2002. The distance between the two distributions in the right part of the support increases, and the density of defaulting firms is much concentrated at negative values. Despite negative performance is experienced also by non-defaulting firms, the evidence suggests that a sort of selection on profitability is at work: default events tend to be associated with lower profitability levels. In addition, time

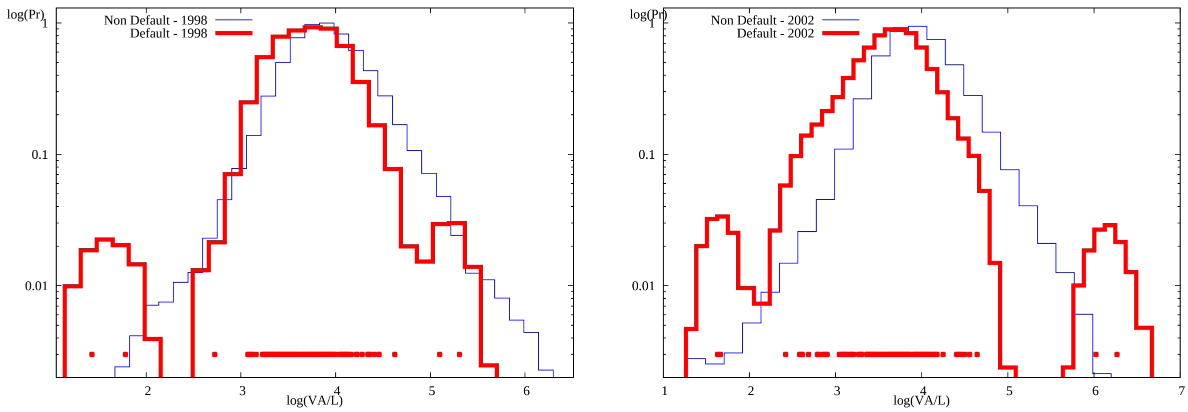

Figure 4: Empirical density of Labour Productivity (VA/L) in 1998 (**left**) and 2002 (**right**): Defaulting *vs* Non Defaulting firms.

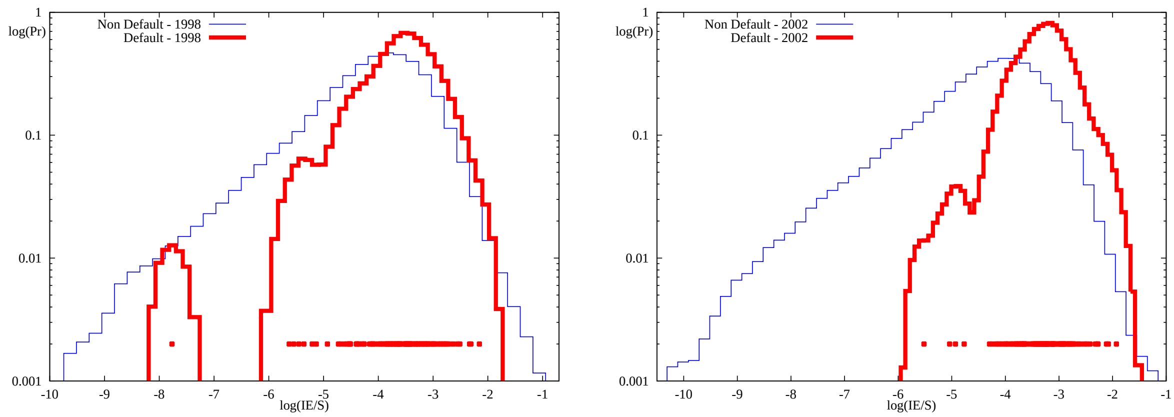

Figure 5: Empirical density of Interest Expenses scaled by size (IE/S) in 1998 (**left**) and 2002 (**right**): Defaulting *vs* Non-Defaulting firms.

plays an important role in the story, since profitability differentials across the two groups tend to become wider in the very short run before default.

Finally, the densities of Labour Productivity, plotted in Figure 4, show that a similar mechanism is also acting upon productive efficiency. The estimates obtained for non-defaulting firms tend indeed to lie above the ones obtained for defaulting firms in the right part of the supports, especially if one nets out the effect of few outliers present at the extremes. The intertemporal patterns also resembles the findings observed for profitability: the productivity advantage of non-defaulting firms increases over time. This suggests that Labour Productivity too represents a discriminatory factor telling apart defaulting firms from the rest of the sample. The relevance of this factor seems increasing as the default event approaches.

3.2 Financial characteristics

We then ask if defaulting firms display any significant peculiarity in terms of the financial variables considered in the analysis: Interest Expenses (IE), Leverage and Financial Debt-

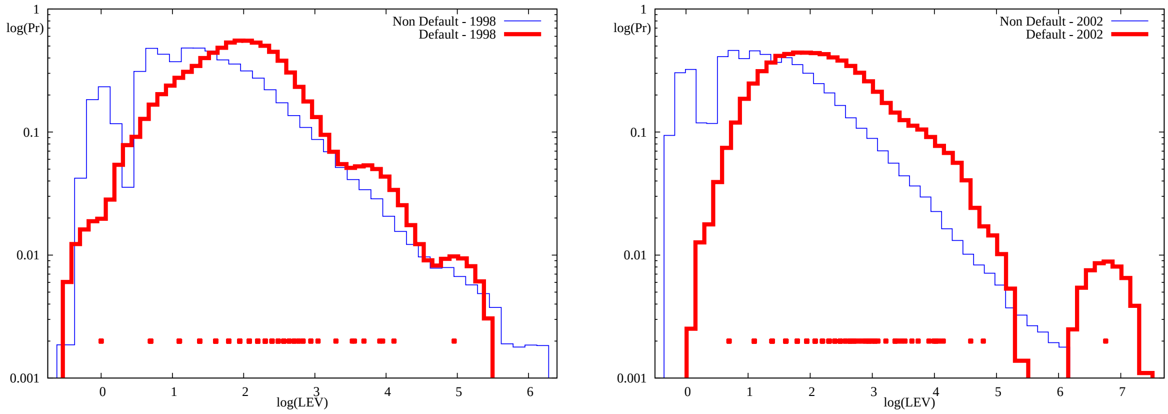

Figure 6: Empirical density of Leverage (LEV) in 1998 (**left**) and 2002 (**right**): Defaulting *vs* Non-Defaulting firms .

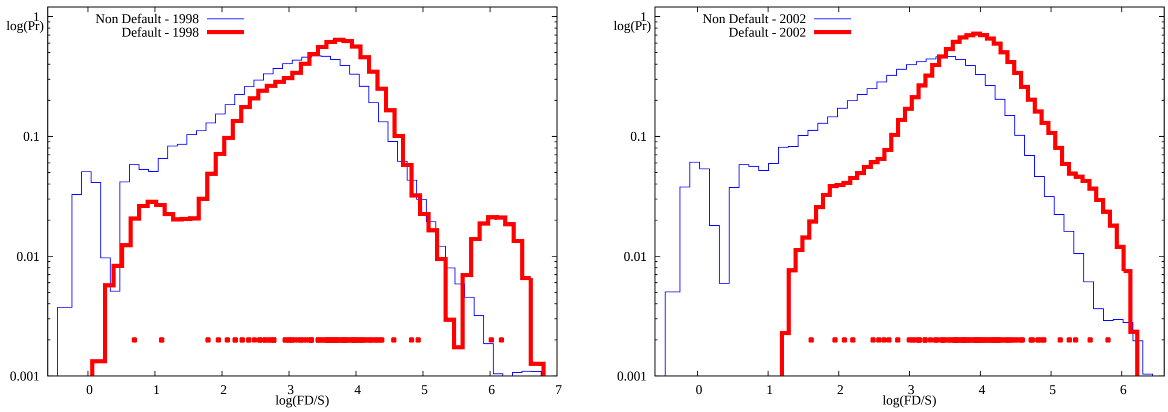

Figure 7: Empirical density of Debt-to-Sales ratio (FD/S) in 1998 (**left**) and 2002 (**right**): Defaulting *vs* Non-Defaulting firms.

to-Sales ratio (FD/S). Notice that the estimates of kernel densities provide information on properties of the variables – such as shape, degrees of heterogeneity among different classes of firms, skewness, etc. – which are usually ignored by financial studies specialized in default prediction.

Figure 5 shows densities of Interest Expenses over Sales IE/S, i.e. the proportion of annual revenues that goes to meet interest payments. The resulting estimates, reported in logs, suggest a clearcut difference between defaulting and non-defaulting firms. Both the average and the modal values of the former group are indeed larger than the ones of the latter. Also the shape of the distributions differ, with the defaulting firms much more concentrated in the right part of the support. A further noticeable feature is that, whereas the estimates for non-defaulting firms do not change over time, the density of defaulting firms displays a rightward shift of probability mass between 1998 and 2002. This means that the flows of interest payments per unit of output sold becomes heavier as the default event approaches.

The densities of Leverage (Figure 6) and Financial Debt-to-Sales ratio (Figure 7) follow similar inter-temporal dynamics. The rightward shift in the Leverage distribution of defaulting firms indicates that the ratio between external *vs.* own resources increases over time, resulting into a disproportionate financial structure in proximity of the default event. At the same time, the even more remarkable shift in the FD/S ratio complement the above results on IE/S: not only the flow of debt repayment, but also the stock of debt is increasing when default approaches. Also notice that the differences between defaulting and non-defaulting firms, in terms of both Leveraged and FD/S ratios, are smaller than in terms of IE/S. This possibly signals that cost of debt is the financial factor which more sharply distinguish defaulters from non-defaulters.

4 Non-parametric inferential analysis

In order to add statistical precision to the comparison between the two groups of firms, we now perform formal tests of distributional equality. A range of testing procedures is in principle available. There are however some specific features of our data which must be carefully considered in selecting the most appropriate alternative. First of all, default events are much less frequent than non-defaults, and therefore we need a test which can be applied in the case of two uneven samples. Second, as shown in the previous section, the distributions we are going to compare display clear non-normalities and unequal variances, suggesting that non-parametric tests should be preferred over parametric ones. Further, even within the class of non-parametric tests for comparison of uneven samples, a common feature is to implicitly assume that the samples to be compared only differ for a shift of location, while their distributions possess identical shapes. However, when distributions with different shapes are compared, looking at the relative location of medians, modes or means might no longer be very informative, as the very meaning of these measures changes with the nature of the underlying distribution. Given that equality of shapes is generally violated by our data, as shown by kernel densities, it is appropriate to employ tests which abandon this hypothesis. A better measure of the relative position of the two samples is provided by the idea of stochastic (in)equality.[7]

Let F_D and F_{ND} be the distributions of a given economic or financial variable, for the two samples respectively. Denote with $\mathbf{X}_D \sim F_D$ and $\mathbf{X}_{ND} \sim F_{ND}$ the associated random variables, and with X_D and X_{ND} two respective realizations. The distribution F_D is said to dominate F_{ND} if $\text{Prob}\{X_D > X_{ND}\} > 1/2$. That is, if one randomly selects two firms, one from the D group and one from the ND group, the probability that the latter displays a smaller value of X is more than 1/2, or, in other terms, it has a higher probability to have the smaller value. Now, since

$$\text{Prob}\{X_D > X_{ND}\} = \int dF_D(X)\, F_{ND}(X) \quad , \tag{1}$$

a statistical procedure to assess which of the two distributions dominates can be formulated as a test of

$$H_0 : \int dF_D\, F_{ND} = \frac{1}{2} \quad \text{vs} \quad H_1 : \int dF_D\, F_{ND} \neq \frac{1}{2} \quad . \tag{2}$$

[7]Consider however that, as a robustness check, we also performed the Wilcoxon-Mann-Witney (WMW) test, which is standard way to assess equality of medians under the assumption of equal shapes. Results were consistent with the evidence presented here below.

Test of Stochastic Equality						
Variable	**Test**	1998	1999	2000	2001	2002
IE/S	FP stat	**7.510**	**9.269**	**13.019**	**17.903**	**24.069**
	p-value	0.000	0.000	0.000	0.000	0.000
LEV	FP stat	**8.029**	**10.483**	**12.066**	**13.520**	**15.190**
	p-value	0.000	0.000	0.000	0.000	0.000
FD/S	FP stat	**7.490**	**10.480**	**14.387**	**16.037**	**17.229**
	p-value	0.000	0.000	0.000	0.000	0.000
TS	FP stat	0.364	1.555	**3.988**	**3.466**	**2.426**
	p-value	0.716	0.120	0.000	0.000	0.015
GROWTH	FP stat		0.905	-0.618	-1.133	**-3.927**
	p-value		0.365	0.536	0.257	0.000
PROF	FP stat	**-4.609**	**-7.169**	**-7.186**	**-7.466**	**-11.176**
	p-value	0.000	0.000	0.000	0.000	0.000
PROD	FP stat	**-5.310**	**-7.156**	**-7.167**	**-6.842**	**-8.855**
	p-value	0.000	0.000	0.000	0.000	0.000

Table 2: Fligner-Policello Test of stochastic equality, Defaulting *vs* Non-Defaulting firms. Observed value of the statistic (FP) and associated p-value. Rejection of the null means that the two distributions are different in probability. Rejection at 5% confidence level highlighted with bold.

The quantity $\hat{U}$ proposed in Fligner and Policello II (1981) provides a valid statistic for H_0. We apply their procedure exploiting the fact that, in case of rejection of the null, the sign of the Fligner-Policello (FP) statistic tells which of the two group is dominant: a negative (positive) sign means that defaulting (non-defaulting) firms have a higher probability to take on smaller values of a given financial or economic variable.[8]

Table 2 presents the results obtained year by year. The high rate of rejection of H_0 supports the evidence provided by the previous descriptive analysis, confirming that the two groups differ under many respects. First, looking at financial variables, the signs of the FP statistics are consistent with the idea that defaulting firms present weaker performances than non-defaulters, under all the dimensions considered. Second, as far as economic variables are concerned, defaulting firms tend to be less profitable and less productive than those in the other group, and we tend to confirm that, possibly due to the already mentioned tail behavior observed in the empirical distribution of Total Sales, defaulting firms are comparatively bigger. Growth rates instead display statistically significant differences in 2002 only.

Overall, the findings broadly confirm the conclusions based on the kernel estimates. Notice also that differences in both economic and financial performances matter over both the shorter and the longer run. With the exception of growth, the null is already rejected at the beginning of the period, or at least some years before default.

[8]Under the further assumption that the two compared distributions are symmetric, testing H_0 is equivalent to testing for equality of medians between possibly heteroskedastic samples. This is what is usually referred to as the Fligner-Policello test.

5 Robust probit analysis of default probabilities

The analyses conducted so far tell us how defaulting firms compare with non-defaulting firms when each economic or financial dimension is considered on its own. In this section we try to identify which are the main determinants of default once the effects of economic and financial factors are allowed to simultaneously interplay.

To this end, we frame our research questions so as to single out the effects of financial and economic variables within a more standard parametric setting. The response probability of observing the default event is modeled as a binary outcome Y (taking value 1 if default occurs, 0 otherwise), and then estimated conditional upon a set X of explanatory variables and controls. We employ a probit model, where the default probability is assumed to depend upon the covariates X only through a linear combination of the latter, $X\beta$, which is in turn mapped into the response probability through

$$\text{Prob}\,(Y = 1 \mid X) = \Phi(X\beta) \quad , \tag{3}$$

where $\Phi(\cdot)$ is the cumulative distribution function of a standard normal variable, with associated density $\phi(\cdot)$.

Several variations of equation (3) are explored in the following, including different sets of regressors. The estimation strategy is however common to all the specifications, and is intended to solve the under-weighting of default events in our data, as compared to distress rates observed in the reference population of Italian limited liability firms (recall Table 1). As anticipated, this feature of the dataset is dangerous for regression analysis, since it might give rise to a classical "choice-based sample" bias (Manski and McFadden, 1981), a well known problem in studies of default probability since Zmijevski (1984). There exist different methods to get rid of the potential bias, either by employing specific estimators designed for this situation (see Manski and Lerman, 1977; Imbens, 1992; Cosslet, 1993), or by performing bootstrap sampling. We follow this second alternative, which has the advantage that it does not depend on specific assumptions about the distribution of the estimated parameters. The only requirement is that each bootstrap sample needs to be representative of the population. Studies of distress prediction, where oversampling of default events is the typical situation, achieve this goal by performing randomized re-sampling of both defaulting and non defaulting firms in the desired, population-wide proportions (see, for instance Grunert et al., 2005). In our case, the relatively low number of defaults available in the data suggests to take defaulting firms fixed, and randomly extract a subset of non-defaulting firms only. This is the strategy we apply in the following. In particular, in order to reduce the bias as much as possible, sampling of non-defaulting firms is implemented with replacement within each 2-Digit industry, so that the ratio of defaulting over non-defaulting firms equals the population-wide default frequency reported in Table 1 at this level of sectoral aggregation. The sampling procedure is repeated several times, and estimates of the different specifications of equation (3) are repeated on the sample obtained at each round. Averaging over the number of runs then yields robust estimates. We will present results based on 200 independent replications, which turned out to be a large enough bootstrap sample to achieve convergence in the estimated coefficients.

One problem remaining out of our direct control concerns the fact, due to the way data are collected, some of the firms treated as non-defaulters could in fact be defaulting firms. Two considerations are due here. First, the possible presence of defaulting into our control group of non-defaulters implies that, whenever a variable has a statistically different effect between the two groups, the "true" difference would be even more significant if we could precisely identify non-defaulters. Thus, we can safely comment on our results when a variable turns significant.[9] On the other hand, it could be that variables that do not turn out to have significantly different effects between the two groups, have indeed different effects, but such differences have been made invisible by the presence of defaulting firms in the control group. Here is where our re-sampling scheme really helps. Indeed, remember that defaults occur with low frequency in the reference population. Therefore, the probability to have a defaulter in the control group number must be very small, and Monte Carlo methods are well known to be robust with respect to this kind of disturbance. So even the occurrence of this second problem can be considered remote.

Our main goal is to test the commonly held presumption that default is mainly determined by poor financial conditions, especially in the short run before default occurs. Thus, our choice of the specifications of equation (3) is primarily meant to verify whether adding economic variables, in general, and looking at their effect at different time distances to default, in particular, might improve the chance to correctly distinguish "healthy" firms from those at risk of default. Our conjecture is that explicit consideration of economic variables should improve the understanding of default dynamics.

Accordingly, we focus on comparing results of two main specifications. The first model includes, among the regressors, only financial indicators, together with a full set of sectoral (2-Digit) control dummies

$$\text{Prob}\,(Y_T = 1 \mid X_t) = \Phi(\beta_{0t} + \beta_{1t}\,\frac{IE_t}{S_t} + \beta_{2t}\,LEV_t + \beta_{3t}\,\frac{FD_t}{S_t} + \delta_t\,\text{Sector}_t) \quad , \tag{4}$$

where, as in the previous sections, IE/S stands for Interest Expenses scaled by Total Sales S, LEV is Leverage, and FD/S is the Financial-Debt-to-Sales ratio. In the second specification we then add the economic variables

$$\begin{aligned} \text{Prob}\,(Y_T = 1 \mid X_t) \quad = \quad & \Phi(\beta_{0t} + \beta_{1t}\,\frac{IE_t}{S_t} + \beta_{2t}\,LEV_t + \beta_{3t}\,\frac{FD_t}{S_t} + \beta_{4t}\,\ln S_t + \\ & \beta_{5t}\,PROD_t + \beta_{6t}\,PROF_t + \beta_{7t}\,GROWTH_t + \delta_t\,\text{Sector}_t) \quad , \end{aligned} \tag{5}$$

where S is size (again in terms of Total Sales), PROD is Labour Productivity (as Value Added per employee), PROF is profitability (in terms of Return on Sales), and GROWTH is the log-difference of Total Sales.[10] Recall that, due to the characteristics of the dataset, the covariates can be measured over the different years of the window 1999-2002, while the default/non default event Y is only measured at the end of the period (at time labeled as T).[11]. Thus, comparing estimates in the different years allows to capture the dynamic effects of the covariates on the probability of default at different time distances to the default event. This is a relevant issue, especially in understanding the extent to which financial conditions are indeed embedding the past history of economic dimensions of firm performance.

[9]This is standard in controlled experiments. Consider for instance that you want to test if a given drug is effective. You treat a group of people for one month and then compare the result with an untreated group. Suppose you find significant differences, and therefore conclude that the drug is actually effective. Now if somebody in the control group had some doses of the drug, this of course testify *in favour* of the effectiveness of the drug, not against it: those control subjects who were not in contact with the drug were different enough to suggest an effective treatment. Coming back to our problem: if we find significant differences comparing the characteristics of defaulters and the control group of non-defaulters, then these differences would be *even more significant* if we could eliminate defaulters from the control group.

[10]Size enters in logs, to reduce the possibly distorting impact of the highly skewed nature of this variable. This is less relevant with other regressors, as they are defined as ratios.

[11]Once again, 1998 is excluded simply because growth rates cannot be computed for that year.

In Panel A of Table 3 we show results obtained in each year, averaging over the 200 bootstrap replications. Columns 1-4 concern estimates of model (4), wherein financial factors alone are considered, while Columns 5-8 refer to the probit specification in (5), where economic variables are added. Notice that all models are estimated taking z-scores of the covariates. This reduces them to have equal (zero) mean and equal (unitary) variance, allowing for a direct comparison of the magnitudes of the estimated effects across different models. We report marginal effects, computed as standard in the sample mean of the covariates, which is zero given z-scoring. Statistical significance is assessed through confidence intervals based on bootstrap percentiles (see Efron and Tibshirani, 1993). That is, we first estimate the empirical probability distribution function (EDF) of the 200 coefficients obtained over the bootstrap runs. Then, statistical significance at the $\alpha\%$ level is rejected if the zero falls within an interval

$$[\hat{q}(\alpha/2), \hat{q}(1-\alpha/2)] \quad , \tag{6}$$

where $\hat{q}(\alpha)$ stands for the estimate of α-th quantile of the bootstrap distribution estimated from the EDF.[12] Estimates of sectoral dummies are not reported, as we indeed find that only less than 5% of these coefficients turns statistically significant at the 5% confidence level. Moreover, the few significant sectors tend to differ across the different exercises considered. These results yield strong support that sectoral specificities do not affect the link between the probability of default and the set of economic and financial characteristics included in our analysis.

The estimates corresponding to the “financial variable only” equation (4) show that cost of debt is the most relevant financial dimension. We indeed find that the relatively big and positive effect of IE/S is significant over the entire period, while Leverage and FD/S display weaker significance. This relates to the interesting variation over time of the estimates. The stock of debt tends indeed to be more relevant at longer distance to default, then loosing significance in the shorter run, when the estimated impact of the IE/S increase remarkably. Notice also that Leverage is turning significant only in the last year before default. This possibly captures part of the short run effect played by an excessive debt burden, thereby compensating for the disappearing significance of FD/S.

The findings in the right part of the table, obtained from specification (5), confirms the predominant role played by cost of debt among the financial indicators, but also offer strong support to the idea that economic characteristics of firms have a relevant effect, additional to that of financial variables. Concerning their sign, the effects, *when significant*, are consistent with the foregoing evidence on kernel densities and stochastic dominance, discussed in Section 3 and Section 4. Size and Growth have indeed a positive effect, while Productivity and Profitability reduce the probability of default. Also notice intertemporal variation of the effects, revealed by varying magnitude of estimates, together with patterns of significance. In particular, Size is strongly significant in all years and the effect seems increasing over time. The marginal effect of Productivity is instead decreasing over time, and looses its significance in the last year. There might be an interaction with Profitability, which indeed turns significant, and with a relatively big negative coefficient, in 2002, possibly “absorbing” part of

[12]Several refinements of the bootstrap estimates of confidence intervals are discussed in the literature, most notably the BC_a and ABC corrections. These methods require an estimate of the bias, which we can only obtain by performing a “first step” probit regression on the overall original sample. This is however exactly what we want to avoid, in order to overcome under-sampling of defaulting firms. Alternatively, one could try to estimate the bias by re-sampling from each random sample. This second order bootstrap seems to us unnecessary due to the relatively large size of the sample considered.

	(1) 1999	(2) 2000	(3) 2001	(4) 2002	(5) 1999	(6) 2000	(7) 2001	(8) 2002
Bootstrapped Probit Regressions - estimates by year								
Panel A: Estimation results								
IE/S	0.0051*	0.0056*	0.0073*	0.0139*	0.0054*	0.0055*	0.0071*	0.0130*
LEV	0.0039	0.0032	0.0026	0.0063*	0.0026	0.0020	0.0014	0.0057*
FD/S	0.0067*	0.0080	0.0064	0.0053	0.0049	0.0072	0.0048	0.0034
ln SIZE					0.0060*	0.0076*	0.0099*	0.0097*
PROD					-0.0128*	-0.0093*	-0.0072*	-0.0023
PROF					-0.0012	-0.0015	-0.0040	-0.0068*
GROWTH					0.0062*	0.0009	-0.0022	-0.0045
CONSTANT	-0.4678*	-0.4684*	-0.4686*	-0.4729*	-0.4706*	-0.4703*	-0.4714*	-0.4761*
Panel B: Model performance								
Brier Score	0.0333	0.0336	0.0336	0.0332	0.0330	0.0334	0.0335	0.0330
Threshold	0.0313	0.0321	0.0320	0.0300	0.0333	0.0323	0.0324	0.0353
Type I error	30.3500	31.9300	23.1650	18.2950	36.5700	29.0950	27.9700	25.3400
Type II error	1541.61	1425.8000	1454.2050	1137.6650	1322.4950	1466.0850	1425.7400	886.7400
% Correct default	0.7629	0.7635	0.8333	0.8603	0.7143	0.7845	0.7988	0.8066
% Correct non default	0.5712	0.6209	0.6250	0.6880	0.6321	0.6102	0.6324	0.7568
Panel C: Model performance								
Threshold					0.0313	0.0321	0.0320	0.0300
Type I error					29.1150	28.5450	26.7150	17.4650
Type II error					1542.1300	1483.1800	1459.9450	1152.6000
% Correct default					0.7725	0.7886	0.8078	0.8667
% Correct non default					0.5710	0.6056	0.6235	0.6839

Table 3: Probit estimates of default probabilities as modeled in Eq. (4) and Eq. (5) – results over 200 bootstrap replications. Variables are in z-scores. Panel A: Bootstrap means of marginal effects at the sample average of covariates, * Significant at 1% level. Panel B-C: Bootstrap means of model performance measures.

the loosing significance of Productivity in this same year. The role of Growth seems instead marginal, with a moderate effect significant only in the first year.

Overall, we can conclude that statistical relevance of economic variables is preserved even at very short time distance to default. Financial and economic dimensions of firm operation, in other words, both matter at longer as well as at shorter run. This suggests existence of strong capital market imperfections, preventing to consider financial structure of firms as embedding all the relevant information on the probability that a firm incur default. Rather, industrial characteristics and performances capture important and complementary determinants of default, even in the short run. The bootstrap procedure, with its random sampling of firms, allows to safely conclude that the observed intertemporal variation cannot be simply attributed to outliers or missing observations affecting the estimates in each specific year.[13]

A further test of the contribution offered by economic variables is conducted in Panel B of the same Table 3, where we focus on goodness of fit and prediction accuracy of the models.

The first measure adopted is the Brier Score (Brier, 1950), a standard indicator employed in the literature to assess the relative explanatory power of alternative models of distress prediction. For each firm i, this is computed as $1/N \sum_{i=1}^{N}(Y_i - P_i)^2$, where N is the number

[13]Also notice that main results persist if we perform bootstrap estimate of linear probability or logit models, and do not change if we take the number of employees as a proxy for size.

of firms, P_i is the estimated probability of default of firm i based on coefficient estimates of the probit regressions, and Y_i is the actual realization of Y for firm i, default or non-default. We report the bootstrap mean of this measure over the 200 replications. Of course, the lower the Brier Score, and the higher the performance of the model. Thus, comparisons of results between corresponding specifications – i.e. looking at each "financial variables only" model *vs.* the "financial plus economic variables" model estimated in the same year – strongly confirm that inclusion of economic variables provides an improved model performance in all years.

Prediction accuracy of the models is then evaluated building upon the concept of correctly classified observations. This is based on the idea of classifying a firm as defaulted (assigning $Y = 1$) whenever its estimated probability of default, P_i, is bigger than a certain threshold value τ, while a firm is classified as non-defaulting otherwise (assigning $Y = 0$). Such a classification will in general differ from the true default or non-default status. A Type I error is defined as the case when a firm which is actually defaulting (a true Y_i=1 in the data) is classified as non-defaulting, and thus is assigned a 0. A Type II error is instead counted when a non-defaulting firm (a true Y_i=0) is assigned a 1 by the classification procedure. Correspondingly, the percentage of correctly predicted 1's ("% Correct default" in the Table) gives the ratio of the correctly predicted defaults over the actual number of defaults in the sample. Conversely, the percentage of correctly predicted 0's ("% Correct non default" in the Table) gives the fraction of correctly classified non-defaulting firms over the actual number of non-defaulters. Within this set of measures, it is standard to prefer models reducing Type I errors (or maximizing the percentage of correctly predicted defaults). Indeed, from the point of view of an investor, failing to predict a bankruptcy (and investing) might be much more costly than mistakenly predicting a default (and not investing).

Quite obviously, the degree of prediction accuracy depends on the specific value of the threshold τ. Different criteria are in principle available to set this value. We consider an "optimal" τ^* so as to minimize the overall number of prediction errors (Type I plus Type II), weighted by the relative frequency of zeros and ones. This is obtained in practice by solving the following minimization problem

$$\tau^* = \arg\min_{\tau} \left(\frac{1}{N_0} \sum_{i \in ND} \Theta(P_i - \tau) + \frac{1}{N_1} \sum_{i \in D} \Theta(\tau - P_i) \right) \quad , \tag{7}$$

where $\Theta(x)$ is the Heaviside step function, taking value $\Theta(x) = 0$ if $x < 0$ and $\Theta(x) = 1$ if $x > 0$, while N_0 and N_1 stand for the actual number of non-defaulting and defaulting firms in our sample, respectively, with ND and D the two corresponding sets.[14]

We repeat the minimization procedure for each bootstrap replication, and then compute Type I and Type II errors, together with the percentage of correctly predicted default and non default events. In Panel B we report averages of these measures computed over the 200 bootstrap replications, together with the average value of the optimal threshold found at each run.[15]

Results suggests that the models including economic variables tend to produce more Type I errors (and thus lower number of Type II errors) as compared to corresponding "financial vari-

[14]Minimizing the overall number of errors is equivalent to maximizing the total sum of correctly predicted observations. The weighting is instead introduced to address the specific characteristics of our exercise. True 0's are indeed much more frequent than true 1's, simply because default rates in each bootstrapped sample equal the population-wide frequencies presented in Table 1.

[15]The application of the bootstrap to compute model performance measures is particularly important. Zmijevski (1984) indeed shows that classification and prediction errors of the defaulting group are generally overstated without an appropriate treatment of the "choice-based sample" problem.

ables only" specifications. It is however important to underline that a direct comparison of these numbers is not truly informative, as they are obtained with different values of τ^*, each optimal for its own model. A much more meaningful comparison between two models would instead require to evaluate the performance of one model under the optimal threshold of the other model. This is done in Panel C of the same Table 3. Here prediction accuracy of the "financial plus economic variables" specifications are computed taking the optimal threshold τ^* of the "financial variables only" model estimated in the same year: if the former performs better under the τ^* of the latter, this would imply a strong confirmation that including economic variables into the analysis is improving predictive power with respect to the benchmark "financial variables only" specification. What we observe is that, compared to the benchmarks figures of Panel B, the models including economic variables perform better in terms of Type I errors (and correctly classified defaults), in all the years but 2001. This offer further evidence that, consistently with suggestions derived from Brier Scores, the contribution of economic variables is important. Once again, improved performance in 2002 confirm that this holds true even in the very proximity of the default event.

In the next section we discuss if such conclusions are robust with respect to inclusion of other variables which, based on previous research in the field, represent major candidates as predictor of default.

6 Robustness checks: including Distance to Default and credit ratings

Following the literature on corporate default prediction, there are two further measures which one should consider in the analysis of default probability, Distance to Default and credit ratings.

Distance to Default is at the core of the last generation of empirical models of default prediction, adopted by both scholars and practitioners.[16] The theoretical foundation of this measure derives from an application of classical finance theory (Black and Scholes, 1973; Merton, 1974), modeling the market value of firm equity as a call option on the value of the firm, with strike price given by the face value of its liabilities. Distance to default (DD) is defined as a function of firms' underlying value of assets, of the volatility of the latter and of the face value of debt. Under the assumptions of the models, the probability of default is completely determined as the value of the density of a normal variable computed in DD, which is therefore considered as a sufficient statistic to predict default. Despite theoretically appealing, DD has two major limitations. First, due to the non trivial estimates required to get a numerical solution of the models, computation of the measure is in practice rather complicated. Second, and relatedly, DD applies to publicly traded firms only, because computation of the underlying values of firms, not observable in practice, is based on the market value of equity, essentially exploiting the standard hypothesis that markets are fully informed and stock prices instantaneously incorporate all information on the underlying value of the firm. A solution to the first problem is to adopt a *naive DD* measure, which is much easier to compute than the original DD and ensures, at the same time, equivalent results in terms of default prediction accuracy (see Bharath and Shumway (2008) where this variant of DD is originally proposed). Yet, the

[16]See Duffie et al. (2007), for the most recent advance in financial literature, and the works cited therein for a review of duration models based on Distance to Default. Crosbie and Bohn (2003) offer an extensive introduction to Moody's KMV model, which is also based on Distance to Default theory.

	Bootstrap Probit with Distance to Default - estimates by year							
	Rating only				Rating, Financial and Economic			
	(1) **1999**	(2) **2000**	(3) **2001**	(4) **2002**	(5) **1999**	(6) **2000**	(7) **2001**	(8) **2002**
Panel A: Estimates								
IE/S					0.0074*	0.0066*	0.0086*	0.0090*
LEV					0.0005	0.0015	0.0002	0.0029
FD/S					0.0024	0.0053	0.0021	0.0026
ln SIZE					0.0066*	0.0076*	0.0075*	0.0072*
PROD					-0.0097*	-0.0067*	-0.0041	-0.0010
PROF					0.0013	-0.0005	-0.0024	-0.0049*
GROWTH					0.0053*	0.0004	-0.0019	-0.0020
Book DD	-0.0210*	-0.0204*	-0.0898*	-0.0816*	-0.0164*	-0.0147*	-0.1095*	-0.1064*
CONSTANT	-0.4698*	-0.4693*	-0.4764*	-0.4766*	-0.4733*	-0.4726*	-0.4800*	-0.4811*
Panel B: Model performance								
Brier Score	0.0332	0.0335	0.0335	0.0335	0.0329	0.0334	0.0333	0.0331
Threshold	0.0374	0.0385	0.0364	0.0362	0.0343	0.0315	0.0345	0.0361
Type I error	43.9900	51.6650	52.2200	49.2350	32.6450	22.6450	30.2400	29.2500
Type II error	1224.4550	1180.9650	1305.3200	1358.4750	1159.7000	1431.3350	1131.6050	901.6500
% Correct default	0.6364	0.5964	0.6014	0.6242	0.7302	0.8231	0.7692	0.7767
% Correct non default	0.6386	0.6677	0.6420	0.6274	0.6577	0.5973	0.6896	0.7527
Panel C: Comparisons of prediction performance against the "Rating only" model of the same year								
Threshold					0.0374	0.0385	0.0364	0.0362
Type I error					41.6750	46.8650	34.7800	29.5750
Type II error					951.9550	974.0700	1020.3050	892.4400
% Correct default					0.6556	0.6339	0.7345	0.7742
% Correct non default					0.7190	0.7259	0.7202	0.7552

Table 4: Probit estimates of default probabilities by year, robustness check to inclusion of *DDBook* as modeled in Eq. (8) and Eq. (9) – results over 200 bootstrap replications. Variables are in z-scores. Panel A: Bootstrap means of marginal effects at the sample average of covariates, * Significant at 1% level. Panel B-C: Bootstrap means of model performance measures.

naive DD still requires data on market values of firms' equity and assets, so that it is not obvious how it is possible to include this measure in the context of our study, where the scope of analysis goes beyond the limited subset of publicly traded firms, and, consequently, only accounting data are available. Nevertheless, motivated by the widespread use and the solid theoretical basis of DD, we attempt to include such a potentially important explanatory variable in the analysis. We start from the *naive DD* estimator of Bharath and Shumway (2008) and build an equivalent measure, denoted *BookDD*, based on accounting data on value of shares and value of debt, which we can derive from available figures on Leverage and Total Assets. Notice that computations exploit time series means and volatility of these variables, and thus *BookDD* takes a single value for each firm, not varying over time (see Appendix I for details on construction of the proxy).

We explore estimates of a first model where *BookDD* enters as the sole covariate

$$P(Y_T = 1 \mid X_t) = \Phi(\beta_0 + \delta \; BookDD) \quad , \tag{8}$$

and then add the full set of financial and economic variables considered in this work

$$P(Y_T = 1 \mid X_t) = \Phi(\beta_{0t} + \delta\ BookDD + \beta_{1t}\ \frac{IE_t}{S_t} + \beta_{2t}\ LEV_t + \beta_{3t}\ \frac{FD_t}{S_t} + \beta_{4t}\ S_t + \beta_{5t}\ PROD_t + \beta_{6t}\ PROF_t + \beta_{7t}\ GROWTH_t) \quad . \tag{9}$$

The estimation strategy goes exactly as in previous section. We resort to bootstrap techniques intended to cure under-weighting of default events in the sample, and evaluate significance levels through confidence intervals based on bootstrap percentiles. Financial and economic variables are taken in z-scores and we perform separate estimates for each year of the sample period 1999-2002.[17]

Results, in Table 4, are clearcut. We obtain that *BookDD* has a tight link with default: marginal effects are big and always statistically significant. However, the inclusion of this further regressor does not affect any of the results achieved in the foregoing section: in all the years considered the sign, magnitude and patterns of statistical significance of the effects of financial and economic variables remain in practice unchanged as compared to our baseline results in Table 3. We only observe that Distance to Default absorbs Leverage and FD/S in some years, but this is not surprising as equity and debts enter the definition of *BookDD*. Moreover, the goodness of fit measure in Panel B show that the inclusion of the *BookDD* does not in general produce a substantial improvement in the predictive ability of the model: Brier score and both types of error are indeed comparable with the ones reported in Table 3. In the same direction, Panel C shows that the inclusion of economic and financial variables yield predictions which are noticeably better that those obtained using only the *BookDD* measure.

A further check including credit ratings has a twofold motivation. First, credit ratings represent, by their very nature, a synthetic measures of dimensions which our set of financial and economic regressors might have only partially captured. Rating procedure are indeed designed to embrace a wide range of firms' characteristics, together with qualitative and quantitative assessment of industry as well as national scenarios, technological changes, regulatory framework, and so on.[18] Second, credit ratings seem also a natural candidate to validate the statistical consistence of the timing effects discovered so far. Indeed, another intrinsic characteristic of credit ratings is their nature of a short-run forecast of default probability, capturing firms' ability to meet their debt positions typically over one year period, or less.

The analysis is performed using the credit rating index developed by CeBi. Our exercise can be replicated taking credit ratings from international agencies, such as Moody's or Standard & Poor's indexes, but a first obvious advantage of the CeBi ratings lies in that they are available for all the firms included in our dataset. On the contrary, international agencies are mainly concerned with bigger Italian firms, those having reached an international relevance, and/or listed on stock exchanges around the world. As a result, using credit files issued by well known rating institutions would bias the scope of analysis towards a sub-sample of firms, not fully representative of the Italian industrial system. Another peculiar characteristic of the CeBi index is that it is an official credit rating. Indeed, founded as an agency of the Bank of Italy in the early 80's, CeBi has a long-standing tradition as an institutional player within the Italian

[17]Statistical irrelevance of sectoral dynamics motivate the exclusion of 2-Digit dummies from the exercise.

[18]This is typically the case with credit ratings issued by international agencies (see the "prototype risk rating system" described in Crouhy et al., 2001). This tendency has been more recently confirmed, by the effect of the provisions of the Basel II process, encouraging banks and financial institutions to also introduce ratings-based internal systems of risk assessment which consider a broad and multidimensional evaluation of their exposure (see BIS, 2001).

		Default year			
		Low	**Mid**	**High**	**Default**
1998	**Low**	0.8948	0.0824	0.0176	0.0052
	Mid	0.5402	0.3651	0.0743	0.0204
	High	0.4589	0.3290	0.1991	0.0130
2002	**Low**	0.9308	0.0583	0.0077	0.0032
	Mid	0.3375	0.5446	0.0986	0.0192
	High	0.1499	0.3512	0.4754	0.0236

Table 5: Credit ratings transition matrices.

financial system. Credit rating construction is one of the core activity within its institutional tasks of providing assistance in banking system supervision. Nowadays a private company, CeBi is still carrying out an institutional mandate, as the Italian member within the European Committee of Central Balance Sheet Data Offices (ECCBSO), operating in close relationships with Italian Statistical Office and the major commercial banks. These observations allow us to be confident that that CeBi ratings are reliable and maintained up to international standards.

On a more detailed ground, the CeBi index is a "issuer credit rating", meaning that it gives an assessment of the obligor's overall capacity to meet its obligations, without implying any specific judgment about the quality of a particular liability of the company. It is updated at the end of each year, and thus allowed to change over time. The method employed for the computation of the index is exclusive property of CeBi. There is however no reasons to expect that the procedure is dramatically different from the methods applied by other rating agencies, both in terms of being targeted over the very short run (as said, one year ahead) and in terms of embracing a wide range of firms' characteristics. The firms included in the database, no matter whether defaulting or non-defaulting at the end of the period, are ranked with a score ranging from 1 to 9, in increasing order of default probability: 1 is attributed to highly solvable firms, while 9 identifies firms displaying a serious risk of default. Notice that the ranking is an ordinal one: firms rated as 9 are not implied to have 9 times the probability of going default as compared to firms rated with a 1.

For the purpose of the present section, we build three classes only, which we label Low Rate firms (having lower probability of default, with credit ratings 1-6), Mid Rate firms (rated 7) and High Rate firms (rated 8-9).

The transition matrices among the three groups, displayed in Table 5, summarize the salient properties of the rating index in the sample. Over the longer-run transition (1998 to default year), the Low Rate class is very stable, while both Mid and High Rate firms display a sort of "reversion to the mean" property, i.e. they have an higher probability to jump back to better ratings, as compared to the likelihood of remaining in the same class. This gets reflected in the transition probability to end up defaulting (last column), which is higher for Mid Rate firms than for High Rate firms. Similar patterns persist in the short run transition (2002 to default year). The numbers on the diagonal suggest higher stability within-class, as compared to the longer-run transition. Yet, the "reversion to the mean" property – towards improved ratings – is still present, even in the High Rate group. Notice however that the short-run transition probabilities to default (last column) are more in accordance with what one would expect: probability of default increases as rating worsens. This confirms the presumption that credit ratings are much better predictors of default in the very short run than over a longer distance to the event. In turn, the fact that the CeBi index displays variation over time is important, allowing to test the time effects of financial and economic variables observed in the foregoing probit regressions.[19]

According to the classification in three rating classes, we build three dummy variables taking on value 1 when a firm is belonging to one of the classes, and zero otherwise. These are then employed in order to investigate if inclusion of different credit rating conditions is able to affect the conclusions drawn from the baseline year by year estimates presented in the previous section. That is, running separate regressions for each year over the 1999-2002 sample period, we first estimate a "rating only" specification

$$\text{Prob}\,(Y_T = 1 \mid X_t) = \Phi(\beta_{0t} + \delta_{1t}\, LOW_t + \delta_{2t}\, MID_t + \delta_{3t}\, HIGH_t) \quad , \tag{10}$$

allowing to get an idea of the explanatory power of the CeBi index, and then compare results with a second model where financial and economic characteristics enter together with ratings themselves

$$\begin{aligned} \text{Prob}\,(Y_T = 1 \mid X) \;=\; & \Phi(\beta_{0t} + \delta_{1t}\, LOW_t + \delta_{2t}\, MID_t + \delta_{3t}\, HIGH_t + \\ & \beta_{1t}\,\frac{IE_t}{S_t} + \beta_{2t}\, LEV_t + \beta_{3t}\,\frac{FD_t}{S_t} + \beta_{4t}\, S_t + \\ & \beta_{5t}\, PROD_t + \beta_{6t}\, PROF_t + \beta_{7t}\, GROWTH_t) \quad . \end{aligned} \tag{11}$$

Table 6 shows the results. Due to obvious collinearity between the rating dummies and the constant term, one dummy cannot be estimated. We present coefficient estimates of regressions where only the Low and Mid Rate class are considered.[20]

Columns 1-4 consider the estimates of regressions where credit ratings are considered alone. The signs and magnitudes of constant term and dummies are consistent with the intertemporal variation of CeBi ratings suggested by transition probability matrices. Low and Mid dummy coefficients essentially depends on the relative proportion of defaulters in these classes, as compared to the proportion of defaulters in the High Rate class (this latter influencing the value of the constant term). The negative estimates of the Low Rate dummy reflects the lower percentage of defaults among Low rate firms, while positive Mid Rate dummy reflects the "reversion to the mean" effect discussed above. Also notice that model performance measures (cfr. Panel B) tend to confirm the 1-year ahead forecast nature of the index: Brier Scores and Type I errors improve approaching the default, and the model of 2002 is the one achieving the best performance records.

The models where we add the other covariates (cfr. columns 5-8) display very similar effects concerning the estimated effects of rating dummies, corroborating the corresponding "rating only" models. More importantly, the effects of financial and economic variables are broadly surviving the inclusion of credit ratings. Credit ratings are surely playing a role,

[19]Notice that firms' "ability" to improve their rating does not depend on the exit of better firms from the sample. The matrices are indeed computed taking all the firms which are still in the sample in the last year, when default is measured, and then tracing back their credit rating history. The findings reported in Bottazzi et al. (2008) show that a similar intertemporal behavior in the CeBi index is also appearing when a different division of firms into rating classes is chosen.

[20]Once again, statistical irrelevance of sectoral dynamics motivate the exclusion of 2-Digit dummies from the models.

Bootstrap Probit with Credit Ratings - estimates by year								
	Rating only				Rating, Financial and Economic			
	(1) **1999**	(2) **2000**	(3) **2001**	(4) **2002**	(5) **1999**	(6) **2000**	(7) **2001**	(8) **2002**
Panel A: Estimates								
IE/S					0.0039*	0.0041*	0.0049	0.0088*
LEV					-0.0001	-0.0013	-0.0015	0.0028*
FD/S					0.0044	0.0059	0.0026	0.0019
ln SIZE					0.0061*	0.0072*	0.0090*	0.0086*
PROD					-0.0111*	-0.0075*	-0.0059*	-0.0011
PROF					-0.0004	-0.0015	-0.0021	-0.0037*
GROWTH					0.0054*	-0.0000	-0.0038*	-0.0036*
CONSTANT	-0.3694*	-0.4770*	-0.3720*	-0.1628*	-0.4999*	-0.6582*	-0.4960*	-0.3021*
LOW	-0.0301*	-0.0076	-0.0323*	-0.1206*	0.0001	0.0186*	-0.0047	-0.0428*
MID	0.0198*	0.0614*	0.0414*	-0.0072	0.0392*	0.1134*	0.0639*	0.0068
Panel B: Model performance								
Brier Score	0.0328	0.0327	0.0322	0.0319	0.0325	0.0326	0.0320	0.0316
Threshold	0.0258	0.0284	0.0193	0.0163	0.0290	0.0254	0.0239	0.0270
Type I error	78.0000	70.7850	61.0700	48.7550	36.6350	30.0450	29.8300	25.8250
Type II error	576.1900	616.3500	709.3900	633.7250	1248.9350	1334.5550	1230.2100	901.1550
% Correct default	0.3906	0.4757	0.5606	0.6278	0.7138	0.7774	0.7854	0.8029
% Correct non default	0.8397	0.8361	0.8171	0.8262	0.6526	0.6452	0.6828	0.7528
Panel C: Comparisons of prediction performance against the "Rating only" model of the same year								
Threshold					0.0258	0.0284	0.0193	0.0163
Type I error					25.2550	39.5450	16.2850	11.8150
Type II error					1612.8600	1111.9150	1722.5850	1507.6900
% Correct default					0.8027	0.7071	0.8828	0.9098
% Correct non default					0.5514	0.7044	0.5558	0.5865

Table 6: Probit estimates of default probabilities, robustness check to inclusion of the credit ratings as modeled in Eq. (10) and Eq. (11) – results over 200 bootstrap replications. Variables are in z-scores. Panel A: Bootstrap means of marginal effects at the sample average of covariates, * Significant at 1% level. Panel B-C: Bootstrap means of model performance measures.

as indeed marginal effects of financial and economic variables are lower as compared to the benchmark estimates without credit dummies (cfr. once again Table 3), but we are able to confirm the main conclusions drawn in the previous section. First, the cost of debt is still playing the main role among financial characteristics, also displaying the expected positive and increasing over time value of the coefficient. Second, economic factors tend to retain their signs. We confirm the positive and strongly significant impact of size, as well as the negative effect of productivity and profitability, with the former significant in first three years, and the latter significant in 2002, exactly as before. The negative and significant role of growth in the last years differ from previous estimates without ratings. This is probably related to instabilities in the tail behavior of this variable across defaulters and non-defaulters, noticed when commenting non parametric distributional properties. Third, and lastly, the findings here reproduce the time effects observed before, in terms of the steady interplay between the different dimensions considered over time. Economic variables, indeed, are as a whole confirmed to be play an important role predictor of default, additional to financial conditions, over both the longer and shorter run.

The main effect induced by including credit ratings seems to be on model performance (cfr. Panel B). As compared to models where credit ratings were not included (compare again with Table 3) we observe lower Brier Scores. These improvements, however, are not entirely accounted for by ratings alone. Indeed, models with the full set of regressors (rate dummies plus economic and financial variables) tend to perform better than the corresponding "rating only" models: when we compute prediction errors taking the optimal τ^* of the corresponding "rating only" models in the same year (see Panel C), Brier Scores are lower and Type I errors decrease considerably.

Summarizing, the analyses of this section validate the robustness of our findings. Both Distance to Default and credit ratings variables can surely complement for some of the relevant dimensions which were not considered in the previous section. Nevertheless, the effects exerted by the set of financial and economic characteristics remain valid, and we can confirm the complementary and additional statistical relevance of economic variables over both the longer and shorter time horizons allowed by the dataset.

7 Conclusion

Firms' distress is considered, in its essence, a financial phenomenon due to firm's inability to repay commercial and financial debts. As a consequence, it is commonly accepted that financial factors should be able, at least to a major extent, to capture the main determinants of default, in particular when the time of the event is approaching. In this paper we show, on the contrary, that industrial or economic characteristics of firms, like size, growth, productivity and profitability, do play a relevant role as determinant of default.

Directly comparing the empirical distributions of a sample of economic and financial variables, we show that firms experiencing default are more financially exposed, less productive and less profitable in all the years before the default occurs, while differences in size and growth suggest a positive relationship with default, although distributional differences tend to be less marked and mainly related with tail behavior of these two variables. Formal tests of distributional equality validate the findings for all the years covered, also adding more reliable statistical support to the evidence on the positive association of size with probability of default (while the role of growth remains negligible). Overall, distributional properties allow us to conclude that it is possible to discriminate will-be defaulting from non-defaulting firms not only on the grounds of their financial situation, but also with respect to their industrial performances, at different time distances to default.

We then show that financial and economic dimensions do not explain the same aspects of default. Rather, the two sets of variables capture diverse, albeit complementary, determinants of the process leading to firms' distress. Indeed, analyzing their respective effects within a set of probit models of default probability, we find that cost of debt exerts the most important effect among the financial variables, but economic characteristics also play a role, which is remarkably significant over the entire time horizon covered by our data. The sign of the estimated effects turns as expected negative for productivity and profitability, which interplay in reducing the likelihood of default, with the latter more relevant very close to default and the former significant in past years. Size displays a positive impact, relatively big and persistent over time. This latter result, consistent with distributional properties of the variable, is less intuitive at first, as one would expect that big firms are relatively more stable than small firms. Part of the explanation for our finding can be attributed to data characteristics, as we record default events only for firms having established a formal credit relationship with a large commercial bank: as indeed suggested by right tail behaviour of size densities, these firms might be relatively big. Notice however that we do not observe over-representation of small firms in the non-defaulting group, suggesting that the threshold imposed on sales to clean the initial sample is not affecting the results.

What is more, probit regressions also provide evidence of interesting time effects: economic variables indeed tend to exhibit strong statistical significance both further away from the default event and in the very short run before default occurs, when financial variables are usually conceived to be more important. As a consequence, the increase in the explanatory and predictive power of the model due to the inclusion of economic variables do not vanishes in the short run.

Robustness of findings is very strong. First, the bootstrap procedure employed should warrant us that probit estimates are not affected by choice-based sampling bias possibly due to under-weighting of defaulting firms as compared to the actual default at the national level. Second, coefficient estimates and time effects survive to inclusion in the models of a Distance to Default and credit rating indexes. This latter result is quite remarkable, since credit ratings represent, by construction, a short-run prediction of default, plausibly embedding many dimensions which are not completely measured by our set of financial and economic regressors. Finally, we find that sectoral dynamics (at the 2-Digit level) do not display any impact: although firms are certainly heterogeneous with respect to both financial and economic characteristics, sectoral specificities do not affect the link between such characteristics and the default probability.

Overall, the findings yield empirical validation to the intuition, so far untested, that default cannot be regarded as a mere financial phenomenon. Rather, our evidence shows that financial indicators do not completely reflect the industrial characteristics of the firms, both over the longer and the shorter run. Clearly, this points toward the existence of severe frictions and capital markets imperfections which create a wedge between financial and industrial characteristics of firms. This result, besides confirming our attempt to link financial and industrial economic research, might have important policy implications. One can indeed suggest that the accuracy of standard risk assessment devices – such as official credit ratings or risk management procedures internally maintained by financial institutions – might possibly devote too few attention to some important, economic rather than financial, factors. Such a tendency is one of the factors giving rise to excessive financiarization and short-termims which can be, and has been, invoked as one of the cause of the deep international crisis started in 2007. Related to this current situation, our results support the necessity to develop broader, multi-dimensional assessments of corporate default risk, placing specific concern to the dimensions included and to the different time horizons considered. At the same time, concerning the way one views the interaction between firms and financial markets, our results give support to measures encouraging the diffusion of an objective and comprehensive approach to evaluate investment decisions and exposure of financial institutions, in line with the original principles underlying the Basel II process, and underline a yet unsatisfied need to develop reliable institutional devices enlarging financial markets' supervision and regulation.

Appendix: costruction of Distance to Default

As explained in the text, we start from the *naive DD* of Bharath and Shumway (2008). For each firm, this is defined as

$$\text{naive DD} = \frac{\ln[(E+F)/F] + (r_{i,t-1} - 0.5 \text{ naive } \sigma_V^2)T}{\text{naive } \sigma_V \sqrt{T}} \quad , \tag{12}$$

where E is market value of equity, F is face value of debt, T is the time-to-maturity assuming each firm has issued just one bond maturing in T periods, $r_{i,t-1}$ is the firm's stock return over the previous year, and σ_V is the volatility of the value of the firms, computed as

$$\text{naive } \sigma_V = \frac{E}{E+F}\sigma_E + \frac{F}{E+F}(0.05 + 0.25\sigma_E) \quad , \tag{13}$$

with the last term in parenthesis being a naive estimate of the volatility of firm debt

$$\text{naive } \sigma_F = 0.05 + 0.25\sigma_E \quad . \tag{14}$$

This default predictor involves a computationally easier estimate of the underlying value of a firm as compared to numerical solution of Black-Scholes-Merton's equations, and its predictive power of default has been found to be comparable to that of the original DD measure.

To make this definition operational in the context of our dataset, where most of the firms are not publicly traded, we make the following choices, based on available accounting book variables. First, we place the time of computation in 2002, the last year before default is measured in our data, so that T=1. Second, E is proxied with the sum of annual income after taxes plus face value of outstanding shares, which we define Book Equity, BE. This is simply the denominator of our measure of Leverage, and therefore we can compute it by

$$BE = \text{Total Assets/Leverage} \quad . \tag{15}$$

Third, since Total Assets equals the sum of BE plus the stock of outstanding debt, due to Italian accounting practices, we can proxy F via

$$D = \text{Total Assets} - BE \quad . \tag{16}$$

Fourth, in place of $r_{i,t-1}$, we take the average of the growth rates of Book Equity, μ_{BE}, which we compute over each year of the sample period before default occurs (1999-2002). This smoothing is done to incorporate all available past information, which is what the *naive DD* assumes to be entirely captured by stock returns over the previous year, due to efficient and fully informed stock markets. Fifth, in place of the approximation of debt volatility contained in Equation (14), we directly compute the volatility of D, as the standard deviation of the growth rates of D in each of the years 1999-2002. Finally, the same is done for σ_{BE}, the volatility of Book Equity. Therefore, our "accounting book version" of the *naive DD* becomes

$$\text{BookDD} = \frac{\ln[(BE_{2002} + D_{2002})/D_{2002}] + (\mu_{BE} - 0.5 \text{ naive } \sigma_V^2)}{\text{naive } \sigma_V} \quad , \tag{17}$$

with

$$\text{naive } \sigma_V = \frac{BE}{BE+D}\sigma_{BE} + \frac{D}{BE+D}\sigma_D \quad . \tag{18}$$

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